Dodo Dudley Hill

January 27 1780

THE

# General Gauger:

OR, THE

## *Principles* and *Practice*

OF GAUGING

## *BEER, WINE*, and *MALT*.

CONTAINING

I. Decimal Arithmetick, the Extraction of the Square and Cube-Roots, and the Uſe of them in the Way of *GAUGING*.

II. The Neceſſary Problems in Geometry relating to *Gauging* and *Meaſuring*.

III. The Uſe of the *SLIDING-RULE* for Meaſuring Timber, and other Surfaces and Solids.

IV. The Theory and Practice of GAUGING in all the Varieties, perform'd by *Pen* and *Sliding-Rule*. Each Propoſition being Wrought in *Beer*, *Wine*, and *Malt*.

V. The Conſtruction of the Table of *Segments*, and all the Varieties of Ullaging of Casks.

VI. A Table of *Area's* in Malt-Buſhels, *&c.*

*The Whole laid down more methodically than any Performance of this Nature yet extant.*

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By Mr. *JOHN DOUGHARTY*, Teacher of the Mathematicks at *Worceſter*.

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*The Fourth Edition, Corrected by the Author.*

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Schoolmaster at Farnboroug
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Oxon

# To the Honourable.

*Foot Onſlow,*
*William Strong,*
*George Townſend,* Eſq;
*Philip Ryley,*
Sir *Marmaduke Wyvil*, Knight.
*Edward Noel,*
——*Mountague,* Eſq;

Chief Commiſſioners and Governours for the Management and Receipt of Her Majeſties Revenue of Exciſe, within the Kingdom of *England*, &c.

*Honoured Sirs,*

Having ſome Time to ſpare from my publick Buſineſs, I thought I could not ſpend it better than in compoſing Something for the Edification of young Officers. And having now finiſhed what I did deſign, and being perſuaded into a Reſolution to print it, I thought my ſelf obliged by the Nature of the Subject, humbly to dedicate the ſame to your Honours, as being the only Perſons that ſeem to encourage Induſtry of this Nature; as may be ſeen by the many prudent and ingenious Inſtructions which your Inferiour Officers daily receive, to the great Advancement of Her Majeſties Revenue, your own Everlaſting Honour, and the general Satisfaction of all Subjects, but particularly,

*Your moſt humble Servant,*

John Dougharty.

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# THE PREFACE.

*THO' the Subject I here treat of, has been accurately purſued by Men of Learning and Merit, yet I can't forbear ſaying, that Gaugers (for whoſe Benefit I chiefly write) have long been at a Loſs, for want of a methodical Introduction, ſuited to the Capacities of Beginners, and adapted to the Practice in which they are imploy'd.*

*Moſt Books upon this Subject throw a Man directly into the Menſuration of Solids, without leading him gradually thro' the previous Parts of Knowledge, upon which the Doctrine and Calculation of the Menſuration of Solids is grounded: And by this means it comes to paſs that the general Practice of Gauging is done by Rote, and all that they know of the Matter is* cation of young Officers. And having now finiſhed what I did deſign, and being perſuaded into a Reſolution to print it, I thought my ſelf obliged by the Nature of the Subject, humbly to dedicate the ſame to your Honours, as being the only Perſons that ſeem to encourage Induſtry of this Nature; as may be ſeen by the many prudent and ingenious Inſtructions which your Inferiour Officers daily receive, to the great Advancement of Her Majeſties Revenue, your own Everlaſting Honour, and the general Satisfaction of all Subjects, but particularly,

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*'Twas upon this Conſideration that I chalk'd down the Method here obſerved. I begin with the firſt and ſimpleſt Principles, and ſo riſe gradually to what is built upon 'em. 'Tis acknowledged on all Hands, that the Calculation of Gauging is moſt expeditiouſly and conveniently done by that Sort of Arithmetick which is call'd* Decimal; *and which for Facility and Diſpatch has much the Advantage of the vulgar Fractional Way. For this Reaſon I begin with a regular and plain Account of all the Operations of Decimal Arithmetick; taking care ſtill to apply the Examples to the Subject of a Gauger's Imployment, that he may ſee the Uſe and Neceſſity of this Part of Knowledge in his Way.*

*Next is the Extraction of the Square and Cube Roots, the Uſe of both which is illuſtrated in ſuch Geometrical Problems as lead to, and are the Foundation of, the Menſuration of Solids, of which Gauging is a Part.*

*This done, I conduct my Reader to the Doctrine of Proportions contain'd in the Rule of Three, and ſhew the uſeful and ready Application of it in ſuch Problems as give light to the Practice of Gauging.*

*Having thus prepared my young Gauger, I ſhould have led him directly to the Gauging Part; but that I thought my ſelf obliged to give him to underſtand the Uſe and Practice of a very uſeful Inſtrument, called, the* Sliding-Rule, *which ſhortens the Calculation. This I exemplify and apply to all the Caſes above-mention'd, that the young Scholar may now apply himſelf to the gauging Part, being equally qualified for working the Rules both by Pen and Inſtrument.*

*As for the Practice of Gauging it ſelf, to which all the above-mentioned Parts are but Preliminaries,*

*Preliminaries, I apply it to all the Varieties requiſite; and diſcuſs each Propoſition both by Pen and Sliding-Rule, in its proper Place. For the greater Conveniency of the Reader, and the Illuſtration of the Matter, I clear every Propoſition, by Rule, by Examples, by Operation, and by proper Schemes. The Examples I make uſe of are in Beer, Wine, and Malt; and the Operation is perform'd both by Pen and Sliding-Rule.*

*For the farther Inſtruction and Satisfaction of the young Gauger, I have added at the latter End, a Table of Circles Area's in Malt Buſhels; together with a Table to gauge right-lin'd Figures.*

*Such are the Method, Deſign, and Contents of the Treatiſe I here publiſh: And tho' I cannot be ſo vain as to plead much upon its Merit, yet I hope the Plainneſs and Eaſineſs of the Method, and the natural Application of its Rules, will atone, in ſome Meaſure, for its Imperfections.*

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## ADVERTISEMENT.

WRiting and Arithmetick, Vulgar, Decimal, Inſtrumental, and by the Logarithms, Algebraical and Geometrical: Alſo Trigonometry, both Plain and Spherical, with their Applications in Meaſuring Boards, Glaſs, Land, Timber, Stone, &c. Book-keeping, Gauging, Navigation, *i. e.* Plain, Mercator and Great-Circle Sailing, Dialling, and Aſtronomy, are *Taught* by the Author, at his Houſe in *Worceſter*: Where Youth for more Expedition may be boarded.

THE

# THE
# General GAUGER, &c.

## Of Decimal Arithmetick.

WHO was the Inventer of this Kind of Arithmetick is uncertain; however, the firſt that profeſſedly treated of this Subject, and gave it the Name of *Diſms*, or *Decimals*, was *Simon Stevinus*, as Dr. *Wallis* thought. But whoever it was, 'tis no matter, ſince our Thanks or Praiſe will do him no good; though indeed we ought to have a great Veneration for the very Name of ſuch Perſons whoſe Labours we are the better for.

It is certain that amongſt the ſeveral Inventions of Man, there is not any thing of greater Uſe than this moſt excellent Part of Arithmetick: Without which the moſt uſeful Parts of mixt Mathematicks would become ſo troubleſome, that it would hinder the moſt patient Soul from attempting any thing that is Praiſe-worthy.

An Officer in the Exciſe therefore ought not to be ſatisfied with leſs than the very Grounds and Reaſons thereof, before he dare undertake to do any thing of Buſineſs. I have not inſiſted ſo much hereon as may be expected, yet what is done is very ſufficient to ſo ſmall a Tract as this.

## *Notation of* DECIMALS.

A Decimal Fraction is ſuch a Fraction whoſe Denominator is never expreſſed in Operation, but always known to be 10, 100, 1000, or 10000, *&c.*

And ſuch a Fraction is known from a whole Number, by having a Comma or Point ſet on the Left ſide thereof.

Thus ,57 is a Decimal Fraction, whoſe Denominator is an 100, for the Denominator of a Decimal Fraction is an Unite with as many Cyphers ſet on the Right ſide thereof, as there were Figures or Places in the Decimal given; ſo the Denominator of ,5 is, 10 of ,25 is 100, of ,127 is 1000, and of ,02 is 100, *&c.* And theſe, or any one of the ſame Nature, may be expreſſed and read as a Vulgar Fraction; thus $\frac{5}{10}$ $\frac{25}{100}$ $\frac{127}{1000}$ $\frac{02}{100}$

(Proportion)

*Note*, That as Cyphers placed on the Right Side of whole Numbers encreaſe them in a decuple Proportion, ſo does Cyphers ſet on the Left ſide of a Decimal Number decreaſe them in the ſame proportion.

### *EXAMPLE.*

If on the right ſide of 6 you place a Cypher, it will be 60, which is ten times more than it was before; ſo if you place a Cypher before ,6 'tis ,06, which is ten times leſs than before.

Alſo if you ſet Cyphers on the Right Side of Decimals, you'll encreaſe the Value thereof no more than you'd do if you had placed Cyphers on the Left ſide of Integers.

### *EXAMPLE.*

Let ,27 be a Decimal, on the right Side of which if you place 5 Cyphers, 'twill ſtand thus ,2700000, but if we ſet

,27 over its imagin'd Denominator 'tis $\frac{27}{100}$ and ,2700000 over its proper Denominator 'tis $\frac{2700000}{10000000}$; now if we conceive the laſt as a vulgar Fraction given to be reduced into its loweſt Terms, 'tis done by taking as many Cyphers from the Denominator as from the Numerator, or cut ſo many off both, by a daſh of your Pen, and 'tis $\frac{27}{100}\Big|\frac{00000}{00000}$ which ſhews it was not encreaſed by thoſe Cyphers.

## Addition of *Decimals*.

As in Addition of whole Numbers, ſo here only you muſt always remember to ſet Units under Unites, and Primes under Primes, *&c.* then add them as Integers, placing a Comma in the Aggregate, under thoſe in the Sums given.

*EXAMPLE.*

| | *l.* | | *l.* | | *l.* |
|---|---|---|---|---|---|
| | 425,73 | | 42,56 | | 432 ,76 |
| | 362,21 | | 3,2 | | 25 , |
| | 43,56 | | ,57 | | 36 , |
| | 5,23 | | ,362 | | 42 , |
| | 6,547 | | ,0521 | | 33 , |
| Sum | 843,277 | Sum | 46,7441 | | ,37 |
| | | | | Sum | 569,13 |

| *ys.* | *ys.* | *ys.* |
|---|---|---|
| 87,642 | 763,57 | 428,27 |
| ,362 | 36,27 | 362,573 |
| ,353 | 36,27 | 621, |
| ,423 | 62,75 | 321, |
| 2,572 | 62,75 | 423, |
| ,425 | 62,7232 | 23, |
| ,00235 | 1024,3332 | ,5, |
| 91,77935 | | 2183,843 |

In all the preceding Examples you may take notice that the Points in the Summs ſtand juſt under the Points in the Numbers given; which if you make before you begin to add them, the Figures will fall in their proper Places.

## Addition of *Beer* or *Ale* Meaſure.

*Note*, That 34 Gallons (by an Act of the firſt of *William* and *Mary*) is one Barrel both of Beer and Ale; therefore the Firkin is 8½ Gallon, *i. e.* 8. 5.

| | *Bar.* | *F.* | *Gal.* |
|---|---|---|---|
| | 365 | 3 | 8,4 |
| | 325 | 3 | 7,1 |
| | 472 | 2 | 5,3 |
| | 846 | 2 | 5,7 |
| | 325 | 3 | 1,5 |
| | 365 | 2 | 1,7 |
| Sum | 2702 | 2 | 4,2 |
| | ✓ | ✓ | ✓ |

| *Bar.* | *F.* | *Gal.* |
|---|---|---|
| 625 | 2 | 8,1 |
| 57 | 3 | 2,2 |
| 57 | 2 | 3,5 |
| 82 | 3 | 7,4 |
| 87 | 3 | 6,7 |
| 57 | 2 | 5,7 |
| 12 | 3 | 8,0 |
| ✓982 | 2 | 7,6✓ |

| *Bar.* | *F.* | *Gal.* |
|---|---|---|
| 357 | 3 | 5,7 |
| 825 | 2 | 2,6 |
| 843 | 3 | 0,2 |
| 562 | 2 | 0,5 |
| 827 | 3 | 0,8 |
| 423 | 0 | 0,2 |
| 3840 | 2 | 1,5 |
| ✓ | ✓ | ✓ ✓ |

In all theſe Examples, you are for every 8,5 Gallons (in the Gallons and Parts Place) to carry one Firkin to the Line of F, and for every 4 in the Line of F you are to take One to the Barrels. The very Examples anſwered are a ſufficient Explanation.

## Subſtraction of *Decimals.*

The ſame Rule is to be obſerved here as in Addition, *i. e.* in placing your Figures; for if you ſet the Point among the Remainders, under the Points in the Number given to be ſubſtracted; after Units are ſet under Units, and Primes under Primes, *&c.* you'll have the true place thereof.

## EXAMPLE.

|  | *l.* | *l.* | *l.* |
|---|---|---|---|
| From | 25,720 | 92,723 | 62,762 |
| Take | 12,972 | 4,320 | 19,200 |
| Remains | 12,748 ✓ | 88,403 ✓ | 43,562 ✓ |

|  | *Gals.* | *Gals.* | *Gals.* |
|---|---|---|---|
| From | 72,027 | 72,63 | 4, |
| Take | 12, | 1,927 | 3,2657 |
| Remains | 60,027 ✓ | 70,703 ✓ | 0,7343 ✓ |

Ex:

## Subſtraction of *Beer* Meaſure.

|  | *Bar.* | *F.* | *Gal.* | *Bar.* | *F.* | *Gal.* |
|---|---|---|---|---|---|---|
| From | 365 | 2 | 7,5 | 272 | 3 | 5,5 |
| Take | 192 | 3 | 3,6 | 192 | 3 | 7,8 |
| Remains | 172 | 3 | 3,9 ✓ | 079 | 3 | 6,2 ✓ |

|  | *Bar.* | *F.* | *Gal.* | *Bar.* | *F.* | *Gal.* |
|---|---|---|---|---|---|---|
| From | 962 | 3 | 2,5 | 836 | 2 | 2,7 |
| Take | 196 | 3 | 7,2 | 192 | 1 | 8,2 |
| Remains | 765 | 3 | 3,8 ✓ | 644 | 0 | 3,0 ✓ |

Ex:

In the Two laſt Examples, ſeeing you can't take 7,2 Gall. out of 2,5 Gall. nor 8,2 out of 2,7; you muſt borrow One Firkin, *viz.* 8,5 Gall. which add to 2,5 or 2,7, and take 7,2 and 8,2 out of the Sum, and the Remainder in one will be 3,8 and in the other 3,0; then pay what you borrow'd to its proper Place, as you ſee is done in the foregoing Operations; which Method ought to be obſerved in all others of the ſame nature.

8,5
2,5
11,0

## Multiplication of *Decimals.*

### GENERAL RULE.

Multiply as in whole Numbers, and from the Product (towards the Right Hand) cut off ſo many Figures as there are decimal Places in both the Multiplier and Multiplicand; but if the Product conſiſteth not of ſo many Figures (as according to this Rule) you ought to cut off, you muſt ſupply the Defect by prefixing one or more Cyphers.

### EXAMPLES.

| | | | |
|---|---|---|---|
| | 3255, | ,3894 | 3,728 |
| | 3,26 | ,678 | 5,43 |
| | 19530 | 31152 | 11184 |
| | 6510 | 27258 | 14912 |
| | 9765 | 23364 | 18640 |
| **Products** | 10611,30 ✓ | ,2640132 ✓ | 20,24304 ✓ |
| | 2,5623 | ,38602 | ,1235 |
| | ,0235 | ,123 | ,036 |
| | 128115 | 115806 | 7410 |
| | 76869 | 77204 | 3705 |
| | 51246 | 38602 | |
| | ,06021405 ✓ | ,04748046 ✓ | ,0044460 ✓ |

In the three laſt Examples the Product did not conſiſt of ſo many Figures as by the general Rule ought to be cut off, therefore the Cyphers were prefix'd to make them ſo many.

## Diviſion *of Decimals.*

### *GENERAL RULE.*

You muſt always cauſe the Decimal Places in the Dividend to exceed thoſe in the Diviſor by about four or five, by ſetting Cyphers on the Right ſide thereof, *&c.* then after the Work is done, prick off with your Pen from the Right Hand of the Quotient ſo many Figures as is the Exceſs; but if the Quotient conſiſteth not of ſo many Figures as by this Rule you ought to cut off, you muſt prefix ſo many Cyphers as will make 'em up ſo many.

### *EXAMPLES of all the Nine Caſes.*

### CASE I.

To divide a whole Number by a whole Number, ſet any Number of Cyphers behind the Dividend, *&c.*

```
236,) 2562,0000 (10,8559 Quotient
      236                  Exd
      ----
       2020
       1888
       ----
        1320
        1180
        ----
        1400
        1180
        ----
         2200
         2124
         ----
           76 ✓
```

## CASE II.

**To divide a mix'd by a whole Number.**

```
96,)  58,3200  (,6075
      576
      ---
       720
       672          +
       ---
       480
       480
       ---
         0
```

*Note*, The Decimal was but ,32, and the Two Cyphers were added to make the Decimals in the Quotient come nearer the Truth than before it would have done.

## CASE III.

To divide a whole Number by a mix'd.

```
3,27)  325,0000000  (99,38837
       2943
       ----
        3070
        2943
        ----
        1270
         981
         ---
         2890           + 1/2
         2616
         ----
          2740
          2616
          ----
          1240       2)1,00(,50
           981
           ---
          2590
          2289
          ----
           301
```

## CASE IV.

To divide a whole Number by a Decimal Fraction.

,572) 3648, 0000000 (63[illegible]6223 Right DBK
3432
2160
1716
4440
4004
4360
4004
3560
3432
1280
1144
1360
1144
2160
1716
~~1444~~

63776223
,572
127552446
446433561
318881115
~~3647999556~~
3648,0000000

## CASE V.

To divide a mix'd Number by a mix'd Number.

1,25) 325,7000 (260,56
250
757
750
700
625
750
750
0

## CASE VI.

To divide a mix'd Number by a Decimal Fraction.

```
,543)  36,2500000   (66,7587
       3258
       ----
       3670
       3258
       -----
        4120
        3801
        -----
         3190
         2715
         -----
          4750
          4344
          -----
           4060
           3801
           -----
            259
```

## CASE VII.

To divide a Decimal Fraction by a whole Number.

```
27)  ,32000000   (,01185185
      27
      --
      50
      27
      ---
      230
      216
      ---
       140
       135
       ---
        50
        27
        ---
        230
        216
        ---
         140
         135
         ---
           5
```

CASE

## CASE VIII.

To divide a Decimal Fraction by a mix'd Number.

13,26) ,36205627 (,027304
2652
9685
9282
4036
3978
005827
5304
523

## CASE IX.

To divide a Decimal Fraction by a Decimal Fraction.

,162) ,8727587 (5,3873
810
627
486
1415
1296
1198
1134
647
486
161

## Reduction of *Decimals*.

When the Parts of the Integer given are reduced to ſuch a Vulgar Fraction as is proper for the ſame, all ſuch Fractions are reduced to decimal Fractions by the Proportion following.

*That is,*

As the Denominator of that Vulgar Fraction is to its Numerator, ſo is 10, 100, 1000, 10000, or 100000, &c. to their particular Numerators, which Numerator is the Decimal required.

### EXAMPLE I.

What decimal Part of a Barrel of Beer, is Five Gallons?

Becauſe in One Barrel are 34 Gallons, therefore the Vulgar Fraction will be $\frac{5}{34}$.

Then the Proportion to find the Decimal Fraction is as 34 : 5 : : 10000

```
              5
34)     5,0000   (,147 facit.
        34
        160
        136
         240
         238
         002
```

```
34
 5
34) 170 (5 Shill
    170
```

L. S

### EXAMPLE II.

What decimal Part of a Gallon is Two Pints?

In One Gallon are Eight Pints, therefore the Vulgar Fraction will be $\frac{2}{8}$, *i. e.* $\frac{1}{4}$, and the proportion is as 4 : 1 : : 100

```
4)   100   (,25 facit.
     2(0
```

### EXAMPLE III.

```
                                              s.   d.
What decimal Part of a Pound Sterling is     15 : 4 ?
                                             12
                                             34
                                             15
Or what decimal Part of a Pound is          184 pence?
```

In One Pound is 240 Pence, therefore the Vulgar Fraction will be $\frac{184}{240}$, and the Proportion as 240 : 184 : : 100000

184

240.) 184,00000 (,76666 Anſwer

1680

1600

1440

1600

1440

1600

1440

1600

1440

160

15,33332

3,99840

3,99360

Ans Proof

### E X A M P L E IV.

What decimal Part of a Pound Sterling is 3 3 $4\frac{1}{4}$ ?

12

40.

4

163 Farthings.

as 960 : 163 : : 10000 : ,1697

*facit* ,1697.

### E X A M P L E V.

What decimal Part of a Foot is 3 Inches?

The Number of Inches in One Foot is 12, therefore the vulgar Fraction will be $\frac{3}{12}$. *i. e.* $\frac{1}{4}$, and the Proportion is as 4 : 1 : : 100 : ,25.

4,) 1,00 (,25 *facit*

20

When a decimal Fraction is given to find the Value thereof.

### R U L E.

Multiply the Decimal given by ſo many of the next inferior Denomination as will make that Integer of which the given Decimal is a Part, and from the Product cut off ſo many Figures as there were Places in the Decimal given.

### EXAMPLE I.

What is the Value of ,752 of a pound Sterling?

20
15,040 Shillings
12
,480 Pence
4
1,920 Farthings
s. d. qt.
Anſwer 15 : 00 : 1,92

### EXAMPLE II.

What is the Value of ,027 of a pound Sterling?

,027
20
,540 Shillings
12
1080
540
6,480 Pence
4
s. d. qt. 1,920 Farthings
Anſwer 00 : 6 : 1,92

### EXAMPLE III.

What is the Value of ,952 of a Pound Sterling?

20
19,040 Shillings
12
,480 Pence
4
1,920 Farthings

*A more ſhort Way to find the Value of a Pound Sterling.*

*RULE.*

Double the Figure which ſtandeth next the Point in the Decimal given; and if the next Figure towards the Right Hand the aforeſaid is 5 or more, add 1 to the Product; then what Figure ſtands in the ſecond Place, above or under 5, reckon ſo many Tens of Farthings; and what ſtands in the third Place from the Prick is ſo many Farthings; which, as often as they are above 13, make leſs by 1; or above 39, make leſs by 2.

*EXAMPLE* IV.

What is the Value of ,482 of a Pound Sterling?

9 : 7½ *facit.*
*s. d.*

*EXAMPLE* V.

What is the Value of ,875 of a Hundred Weight Averdupoize?

4
3,500 Quarters
28
14.000 Pounds. } Ans.

*EXAMPLE* VI.

What is the Value of ,623 of a Barrel of Beer?

4
2,492 Firkins.
8,5
2460
3936
4,1820 Gallons.
8
1,4560 Pints.

*F. G. P.*
*Facit* 2 : 4 : 1,456

To

## To extract the Square Root.

TO extract the Square Root of any given Number, is to find a Number, which multiplied into it ſelf ſhall produce the Number given; and this is of great Uſe in Gauging, for finding mean Proportionals, *&c.*

When a Number is given, whoſe Square Root is required, you muſt firſt make a Point over every other Figure, beginning at Unity, and ſo many Points as you have, ſo many Figures you muſt expect in the Root required.

*EXAMPLE.*

Let the Square given be 472738, which being pointed (as is above directed) is $4\dot{7}2\dot{7}3\dot{8}$, by which I conclude that the Root will have Three Places, becauſe the Points are Three which ſtand over the given Square.

Your next Buſineſs is to find the Root of the greateſt Square to the firſt Point; which in this Caſe you'll find to be 6, for the Square thereof is 36, that being the next leſſer Square to the firſt Point, *viz.* 47; this 36 I take from 47, and place the Root, *viz.* 6, in the Quotient, and the Work will ſtand thus,

$4\dot{7}2\dot{7}3\dot{8}$ (6
$\underline{36}$
11

To the laſt Remainder I muſt bring down all the Figures in the next Point, and then 'tis

$4\dot{7}2\dot{7}3\dot{8}$ (6
$\underline{36}$
1127

And for a Diviſor I double the Quotient, and 'tis

472738 (6
$\underline{36}$
12) 1127

Then enquire how many times you can have 12 in 112, which you can have but 8 times, ſet your 8 behind the Diviſor, and alſo in the Quotient, and 'twill ſtand thus,

```
      472738 (68
      36
128)  1127
```

Multiply 128 by 8, and ſubſtract the Product from 1127, and the Remainder will be 103, as in the following.

```
      472738 (68
      36
128)  1127
      1024
       103
```

To the laſt Remainder, *viz.* 103, I bring down all in the next Point, *viz.* 38, and double the Root (*viz.* 68) for a Diviſor, and it will ſtand thus,

```
      472738 (68
      36
128)  1127
      1024
136)   10338
```

Then again enquire how many times 136 in 1033 (for 8 is to be reſerv'd for the Square of the Figure that is to be found) which is 7 times; which you muſt place in the Root, and behind the double Root, and the Work is as below.

```
       472738 (687 Root
       36
128)   1127
       1024
1367)  10338
        9569
         769 remains.
```

But if more Exactneſs be required (as to have the Root conſiſt of 6 or 7 Places of Decimals) you may annex 12 or 14 Cyphers to the Square given, and then extract, as by the foregoing Example you are directed, ſo will your Root conſiſt of whole Numbers and Decimals as followeth. *

. . . . . . . . . .

472738,00000000000000 (687,559~~4519~~
36

128) 1127
1024

1367) 10338
9569

13745) 76900
68725

137505) 817500
687525

1375109) 12997500
12375981

13751184) 62151900
~~55004736~~

137511885) ~~714716400~~
~~687559425~~

1375118901) ~~2715697500~~
~~1375118901~~

13751189029) ~~134057859900~~
~~12376070121~~

~~1029715863~~ remains.

Needleſs.

The

---

* *Where Note, That if the Number of decimal Places in the Number given are not even, you ought to make 'em ſo, by putting one Cypher on the right Side before you begin to extract.*

## The Uſe of the *Square Root.*

### PROB. I.

### *To find a mean Proportional between any Two Numbers.*

### *RULE.*

Multiply one of the Numbers given by the other, and the Square Root of that Product is the mean Proportional ſought.

### *EXAMPLE.*

What is the Mean between 46 and 62 ?

46

372
248

2852, (53, 40, &c.
25

103) 352
309

1064) 4300
4256

1068) 4400

62
46

2)108

54.00
53.40

.60
too much

D D Hill

Anſwer 53, 4.

By the foregoing Rule you'll find the Means between

| | | | |
|---|---|---|---|
| 32 & 46 | to be | 38, 36, &c. | Exd DD Hill |
| 82 & 49 | to be | 63, ~~50,~~ 38 &c. | DD Hill |
| 47 & 64 | to be | 54, ~~65,~~ 745 &c. | DD Hill |

## PROB. II.

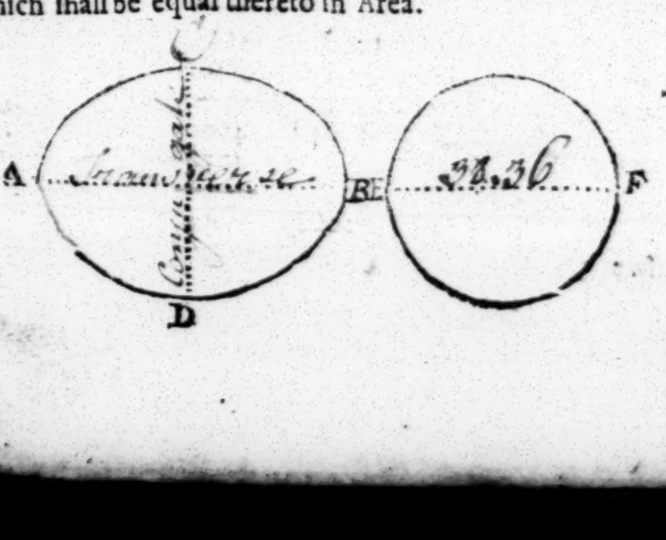
*To find the Diameter of a Circle which ſhall be equal in Area to any Oval, whoſe Conjugate and tranſverſe Diameters are given.*

*RULE.*

Multiply the Conjugate by the tranſverſe Diameter, and extract the Square Root of the Product, which Root is the Diameter of that Circle whoſe Area is equal to that of the Oval.

*EXAMPLE.*

Let AB be the tranſverſe Diameter of an Oval, which let it be 46 Inches, and let CD be the Conjugate equal to 32 Inches, I demand the Diameter EF of that Circle, which ſhall be equal thereto in Area.



46

```
        46  Transverse
        32  Conjugate
        --
        92
       138
      ----
      1472   (38,36 facit.
         9
  68)  572
       544
 763)  2800
       2289
7666)   511,00
```

46
32
2) 78
39.

## PROB. III.

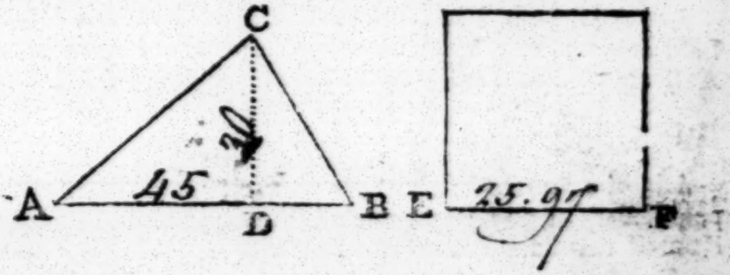
### *To find the Side of a Square* EF, *which ſhall be equal in Area to any given Superficies.*

*RULE.*

Find the Area of the given Superficies, and the Square Root thereof ſhall be the Side of a Square whoſe Area ſhall be equal to that of the Superficies given, whether it be a Circle, Oval, Triangle, Parallelogram, *&c.*

*EXAMPLE.*

There is a Triangle whoſe Baſe AB is 45 Inches, and Perpendicular CD 30: I demand the Side of a Square which ſhall be equal in Area to the ſaid Triangle.



```
           45
           15 half the Perpendicular CD
          ———
          225
           45
          ———
          675,0000 (25,97 the Side EF of the Sq.
          4
         ———
  45)     275
          225
         ————
  509)     5000
           4581
          —————
  5187)    41900
           36309
          ——————
            5591 remains.
```

## PROP. IV.

*Having given (in a Right-angled Triangle) any Two Sides, the Third may be found by the 47th of the firſt of Euclid, where 'tis demonſtrated that the Square of the Hypothenuſe,* viz. BC, *is equal to the Sum of the Squares of* AC *and* AB.

### CASE I.

Let ABC be a Right-angled Triangle, in which AB the Baſe equal to 24 Yards is given, and the Perpendicular AC, equal to 18 Yards; to find the Hypothenuſe BC.

### *RULE.*

Add the Square of the Baſe to the Square of the Perpendicular (of any Right-angled Triangle) and the Square Root of that Sum is the Hypothenuſe of the ſame Triangle.

*EX-*

### EXAMPLE.

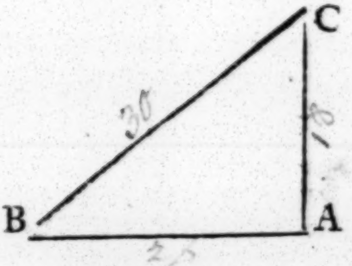
| | |
|---|---|
| AB the Baſe = 24 | AC the Perpendicular 18 |
| 24 | 18 |
| 96 | 144 |
| 48 | 18 |
| its Square 576 | its Square 324 |

324

) 900 (30=BC theHypothenuſe required.

9

3) 000



## PROB. V.

### CASE II.

*Given the Hypothenuſe* BC = 30, *and the Baſe* AB = 24, *to find the Perpendicular* AC.

### RULE.

From the Square of the Hypothenuſe ſubſtract the Square of the Baſe, and the Remainder will be the Square of the Perpendicular, whoſe Square Root will be the Perpendicular it ſelf.

*EXAMPLE.*

| | | | |
|---|---|---|---|
| The Hypothenuſe | 30 | The Baſe | 24 |
| | 30. | | 24 |
| its Square | 900 | | 96 |
| | 576 | | 48 |
| | 324 (18 | | 576 = its Square. |
| | 1 | | |
| 28) | 224 | | |
| | 224 | | |
| | 0 | | |

Anſwer, the Perpendicular AC is 18 Yards.

## PROB. VI.

### CASE III.

*The Hypothenuſe* BC=30, *and the Perpendicular* AC=18, *given, to find the Baſe* AB.

*RULE.*

From the Square of the Hypothenuſe, take the Square of the Perpendicular, then the Square Root of the Remainder, is the Baſe required.

*EXAMPLE.*

| | | | |
|---|---|---|---|
| Hypothenuſe BC= | 30 | Perpendicular | 18 |
| | 30 | | 18 |
| its Square | 900 | | 144 |
| Squ. of the Perp. | 324 | | 18 |
| Remains | 576 | its Square | 324 |
| | 4 | (24 Root | |
| 44) | 176 | | |
| | 176 | | |
| | 0 | | |

Anſwer; 24 is the Length of the Baſe BA required.

PROB.

## PROB. VII.

*Given the ſlanting Side of a Pyramid or Cone, and alſo the Length of a Perpendicular let fall from the Vertex to the Center of the Baſe: To find the Diameter of the Baſe.*

### RULE.

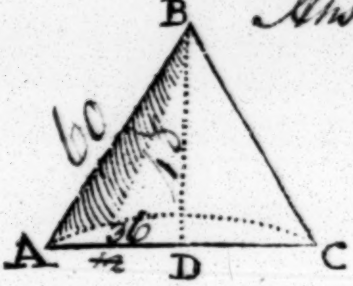
From the Square of the ſlanting Side, ſubſtract the Square of the Perpendicular Line, then twice the Square Root of the Remainder is the Diameter of the Baſe required.

### EXAMPLE.

Let the ſlanting Side be AB=60; which (becauſe ABD is a Right-agled Triangle) may be called the Hypothenuſe; and let the Perpendicular given be BD=48, to find AD = $\frac{1}{2}$ AC.

BD=48
AB=60 48
60 384
192
its Square 3600
BD's Squ. 2304 and 2304 is its Square.
Rem. ) 1296 (36 Root = AD = $\frac{1}{2}$ AC.
9 therefore AC=72 requir'd.
66) 396
396
0

Ans 36 AD ½ AC



## PROB. VIII.

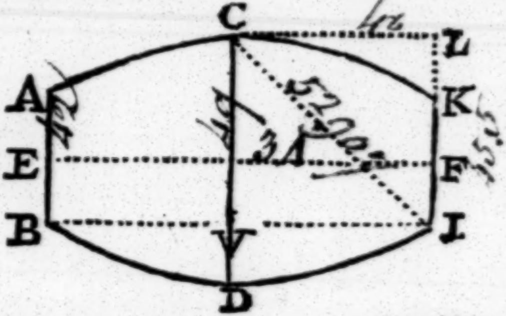
*Given the Bung and Head Diameters (of a Cask,* &c.) *and alſo the Length ; to find the Diagonal Line.*

### RULE.

To the Square of Half the Sum of the Diameters, add the Square of Half the Length, then the Square Root of the Sum is the Diagonal Line required.

### EXAMPLE.

Let the Head Diameter be AB=IK=42 Inches, the Bung Diameter CD=49 Inches, and the Length EF=BI =34; What is the Diagonal Line CI.



*Note*, that = ſignifies Equal to, and + ſignifies to Add or More.

The half Difference between the Head KI, and Bung CD is LK, which if you add to the Head KI, the Sum will be the Leg. LI (of the Right-angled Triangle CLI) equal to Half the Sum of the Diameters, *viz*. CD+KI.

The length is EF=BI; Half of which is VI=CL, the other Leg. of the Right-angled Triangle CLI: By which you may find the Hypothenuſe CI, according to the Caſe of Prob. 4.

LI=45.5 } Given, to find CI.
CL = 27. }

CL=27
27
189
54
CL □ 729

LI=45.5
45.5
2275
2275
1820
2070.25 LI□
729. + □ of CL
2799.25 (52.907 V.)
25
102) 299
204
1049) 9525
9441
105807) 840000
740649
.99351

Cd

## PROB. IX.

### *The Diameters and Diagonal given, to find the Length.*

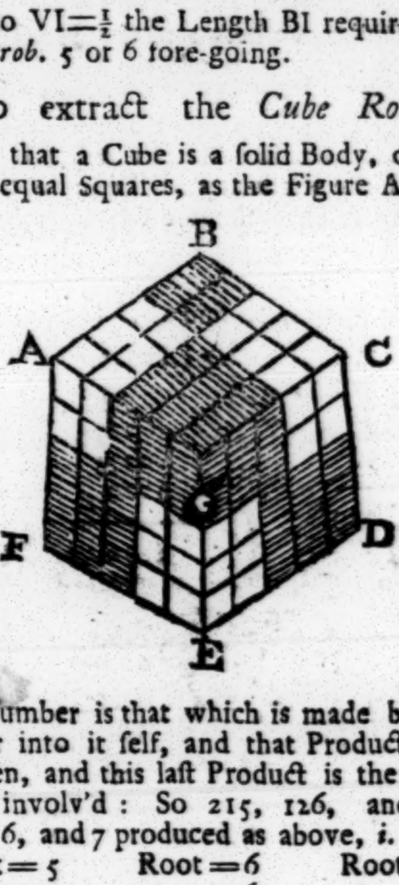
In this Problem you have IL, equal to half the Sum of the Diameters, and the Diagonal Line CI: to find CL, which

will

will be equal to $VI = \frac{1}{2}$ the Length BI required, and may be found by *Prob.* 5 or 6 fore-going.

## To extract the *Cube Root.*

NOTE, that a Cube is a ſolid Body, contained under ſix equal Squares, as the Figure ABCDEFG.



A Cube Number is that which is made by multiplying any Number into it ſelf, and that Product again by the Number given, and this laſt Product is the Cube of the Number ſo involv'd: So 215, 126, and 343 are the Cubes of 5, 6, and 7 produced as above, *i. e.*

Root = 5 Root = 6 Root = 7
5 6 7

its Square = 25 its Square = 36 its Square = 49
5 6 7

its Cube = 125 its Cube = 216 its Cube = 343

By this you may know, that the Product of any Number in it ſelf is called the Square of that Number; which

Square multiplied by its Root, produceth the Cube, or third Power of the ſame Number, *&c.*

To know of how many Figures the Root of any Cube doth conſiſt, you muſt point every third Figure, beginning over Unity, and ſo many Points as you make, ſo many Figures you muſt expect in the Root.

What is the Cube Root of 12812904?

After having pointed every third Figure from Unity, 'twill ſtand thus, $1\dot{2}81\dot{2}90\dot{4}$, and becauſe there are three Points over the given Cube; I muſt therefore expect that the Root ſhould conſiſt of three Places.

### *OPERATION.*

The next greateſt Cube to the firſt Point, or under 12, is 8, whoſe Root is 2: This 2 I muſt place in the Root or Quotient, and take its Cube, *viz.* 8 from 12, and the Work will ſtand thus,

$$\begin{array}{l} 1\dot{2}81\dot{2}90\dot{4}\ \ (2 \\ 8 \\ \hline \end{array}$$

To this 4 which remains 4 I bring down all to the next Point, and the Work will ſtand thus,

$$\begin{array}{l} 1\dot{2}81\dot{2}90\dot{4}\ \ (2 \\ 8 \\ \hline \end{array}$$

And the Reſolvend is 4812

Now for a Diviſor, I muſt ſquare the Quotient or Root 2, and that will be 4; which multiplyed by 300, is 1200; to which add the Rectangle of the Root 2 in 30, and the Sum will be 1260, for a Diviſor.

Where, Note, that the Diviſor is always found by multiplying the Square of the Quotient by 300, and adding that to the Product of the Quotient it ſelf in 30, as above and hereafter may be ſeen.

This Diviſor, *viz.* 1260, I find may be had 3 times in 4812
for 3 in 1200 (the Square of the Quotient in 300) is 3600
and 9 (the Square of the Root 3) in 60 is 540
to which add the Cube of 3, *viz.* 27
and the Sum is 4167
which take from 4812, and there will remain 645 as below.

12812904 (23
8
———
4812
4167
———
Remains 645

To this Remainder I bring down all to the next Point, and 'twill ſtand thus;

12812904 (23
8
———
4812
4167
———
for a Reſolvend 645904

Again, (as before) I ſquare the Quotient or Root, *viz.* 23, and that is 529, which Square muſt be multiplied by 300, and the Quotient it ſelf by 30; then the Sum of theſe two Products is 159390, for a Diviſor; which (by Tryals) I find I may have 4 Times in the Reſolvend 645904; therefore as (above)

$529 \times 300 \times 4 =$ 634800
and $23 \times 30 \times 16 =$ 11040
to which add the Cube of $4 =$ 64
and the Sum is 645904

which take from the laſt Reſolvend, and the Remainder will be 0, and appear as below.

12812904 (234
8

4812
4167

645904
645904

0

Where alſo note, that becauſe nothing remains, the Number given was a true Cube Number.

But if any thing had remain'd, you ſhould have ſet 3 Cyphers on the right Side of the Remainder, and then proceed to find the next Figure to put in the Qotient, as before: Where you muſt alſo note, that for every three Cyphers which you make uſe of this way, you will have one decimal Place in the Root. But before you begin to extract the Cube Root of any Number, you may ſet 3, 6, 9, or 12 Cyphers on the right Side of it, and point all as above directed, and then proceed to find the Root, as in the Example following.

Let it be required to extract the Cube Root of 427362

. . . . . . .
427362,000000000000 (75,3237 Root.
343

84362
78875

5487000
5082777

404223000
340295768

63927232000
51059955267

12867276733000
11914574816053

952701916947 Remains.

If the Number given, to have its Cube-Root extracted, has any Number of decimal Places that are not 3, 6, 9, or 12, *&c.* you muſt make them ſo by placing Cyphers on the right Side, and then point them beginning as in whole Numbers.

## The Uſe of the *Cube Root.*

### *The Diameter and Weight of a Bullet given, to find the Weight of another Bullet, whoſe Diameter is alſo given.*

This is anſwered from the *Prop.* 18. of the 12th Book of *Euclid*; from whence may be drawn the Proportion following, *i. e.*

As the Cube of the Diameter of the given Sphere or Bullet, is to the Cube of the Diameter of any other Sphere or Bullet, ſo is the Weight of that given Bullet, *&c.* to the Weight of the other requir'd.

### EXAMPLE.

There is a Bullet whoſe Diameter is 4 Inches, and its Weight 9 Pounds: I demand the Weight of another Bullet, whoſe Diameter is 7,5 Inches.
the Cube of 4 is 64
and the Cube of 7,5 is 421,875
Then ſay as 64 : 421,875 : : 9*l.* : 59,3261 *l.* thus,

```
                9                lbs
           ─────────
    64)    3796,875      (59,326 Answer 11/64 lbs
           320
           ───
            596
            576
            ───
             208
             192
             ───
              167
              128
              ───
               395
               384
               ───
                11
```

```
     7,5
     7,5
    ────
     375
    525
   ─────
   56,25
     7,5
   ─────
   28125
  39375
  ──────
 421,875
```

### PROB.

## PROB. I.

*There is a Conical Tun, whoſe bottom Diameter is* 40 *Inches, the Diameter at the Top is* 30 *Inches, and the Depth is* 20: *According to which 'twill contain* 68,7 *Beer Gallons. I would know the Diameters and Depth of another Conical Tun, which will hold* 90 *Gallons of Beer.*

The Solution of this Problem may be taken from *Prop.* 36. *Lib. Matheſ. Enucl.* where 'tis demonſtrated that ſimilar or like ſolid Figures, are to one another in triplicate Proportion of their Homologous Sides, *i. e.* as the Cubes of their like Sides.

### *RULE.*

From the above-mentioned *Prop.* we may infer the Propoſition following, *viz.*

As the Content of the given Tun, *viz.* 68,7 is to the Cube of 40 (the bottom Diameter) = 64000; ſo is 90 the Content of the required Tun, to 83842,794; which is the Cube of the bottom Diameter of that Tun whoſe Content is 90, and the Cube Root thereof is 43,77 the Diameter required.

Again, for the top Diameter ſay,

As the Content of the Tun given, *viz.* 68,7 is to the Cube of its top Diameter, *viz.* 30; ſo is 90 the Content of the other Tun, to the Cube of its top Diameter, whoſe Cube Root is 32,82,

Laſtly; for the Depth ſay,

As 68,7 is to the Cube of its Depth, (*viz.* the Cube of 20) ſo is 90 the Content of the other Tun to the Cube of its Depth; whoſe Cube Root is 21,88, the Depth required: To prove which you now have the Diameters and Depth of a Conical Tun; which by the Rules following, you'll find will contain near 90 Gallons, and the like of any other ſuch Figure.

## *To calculate Diagonal Tables or Lines for Casks, taken as the middle Fruſtum of a Spheroid.*

By *Prob.* 8. of the Square Root find the Diagonal thus; let the Bung Diameter be 26 Inches, the Head 20, and Length 24: What's the Content and Diagonal.

| | |
|---|---|
| Bung 26 | Length 24 |
| Head 20 | ½ the Length 12 |
| Sum 46 | 12 |
| ½ the Sum 23 | its Square 144 |

23
69
46
its Square 529
144 add
673,00 (25,94 Diagonal of the Cask.
4
45) 273
225
509) 4800
4581
5184) 21900
20736
1164

The Content of the Veſſel is 39 Beer Gallons.

Having by the Bung, Head, and Length, above given, found theſe two Requiſites, *viz.* the Content of the Cask and its Diagonal, you may find the Diagonal Line of any other Cask, whoſe Content is given by the Proportion following, *viz.* as 39 is to 17454,6, ſo is 1, 2, 3, 4, 5, 6, 7, *&c.* the Contents of another Spheroidal Cask to the Cube of their Diagonals, whoſe Cube Roots will be the Diagonals for the Contents given; which you may collect into a Table or project upon an Inſtrument, as occaſion requires. But here, note, that though I ſhall give one Example here, and perform it according to decimal Arithmetick, for Order's ſake, yet this Sort of Work may be beſt managed by the Logarithms, as you may ſee in Mr. *Marmaduke Hodgeſon*'s Book of Gauging, *Pag.* 67, 68.

### *EXAMPLE.*

Let it be required to find the Length of the Diagonal Line of that ſpheroidal Cask, whoſe Content is 20 Gallons.

Say, as 39 the Content of a ſpheroidal Cask, is to the Cube of its Diagonal Line, 17454,6; ſo is 20 the Content of another ſuch Cask, to 8951, the Cube of its Diagonal, whoſe Cube Root is, 20,7 the Anſwer.

By this you may know, that if you let fall a Rule (having only a Line of Inches on it) Diagonal-wiſe from the Bung, and find that the Length of that Line is 20,7 Inches, that the Content of that Cask is 20 Beer Gallons.

This may be better underſtood, and done with much more Expedition when I come to ſpeak of the Sliding-Rule. And here note, that to find the Side of a Cube which ſhall be equal in Content to any given Solid, is only to extract the Cube Root of the Content of that Solid, whether it be a Cone, Sphere, or Cylinder, *&c.*

The

# The Rule of *THREE.*

I Need not go about to define this Rule, ſince the very Name is a ſufficient Definition: However, ſome Queſtions proper to it are as follow.

## PROB. I.

*c.* *l.* *s.* *l.*
If 42 coſt 172 : 16 : what will 47 coſt?
112 172,8 in a decimal Way.
224 47
448 12096
4704 6912

4704) 8121,6000 (1,7265, *i. e.* by Reduction
4704
*l.* *s.* *d.* *q.*
34176 1 : 14 : 6 : 1,44 the Anſwer.
32928

12480
9408

30720
28224

24960
23520

1440

$\frac{32}{40} = \frac{16}{20} = \frac{8}{10}$ .8

If the Duty of 16 Buſhels of Malt be 8 Shillings; what is the Duty of 732 at that Rate?

*B.* *l.* *B.*
If 16 : ,4 : : 732
,4
*l.* *l.* *s.* *d.*
16) 292,8 (18,3, *i.e.* 18 : 6 : 00. for Anſwer.
134(0
00

If the Duty of 32 Barrels of X Beer be worth
*l.* *s.*
7 : : 12. what's the Duty of 72,56 Barrels?
*B.* *l.* *s.* *l.* *B.*
If 32 give 7 : 12, *i. e.* 7,6; what will 72,56 give?
7,6

43536
50792

32) 551,456 (17,233
23709(0
010(0
*l.* *s.* *d.* *q.* *l.*
Anſ. 17,233; which by Reduction is 17 : 4 : 7 3, 68

If the Sallary of an Officer be 48,5 *per Annum*, what muſt be paid him for 232 Days?

,12
20
2,40
48

If

*d.* *l.*
If 365 give 48,5; what will 232 Days?

232
970
1455
970

365) 11252,000 (30,827, anſwer, *i. e.*
1095

3020 *l.* *s.* *d.* *q.*
2920 30 : 16 : 6 : 1,92
1000
730
2700
2555
145

If the Duty of 172 Buſhels of Malt be
*l.* *s.* *l.*
4 : 6. *i. e.* 4,3; what will the Duty of 92 Buſhels be?

*B.* *l.* *B.*
If 172 give 4,3; what will 92 give?

92
86
387

*l.* *l.* *s.*
172) 395,6 (2,3. *i.e.* 2 : 6. for Anſwer.
344,
516
516
0

20 = 1

If

l. s. l.

If the Duty of 92 Buſhels of Malt be 2 : 6. i. e. 2,3; what is the Duty of 172 Buſhels?

B. l. B.

If 92 give 2,3; what will 172 give?

2,3

516

344

l. s.

92) 395,6 (4,3 anſ. i. e. 4 : 06:

27(0

(0

If the Intereſt of 100 Pounds be 5 *l. per Annum*; what may I demand for the Uſe of 45 *l.* : 18 *s.* ; *i. e.* 45,9 *l.* for one Year at that Rate?

*l.* *int.* *l.*

If 100 give 5; what 45,9

5

*l.* *l.* *s.* *d.* *q.*

100) 229,5 (2,295, i. e. 2 : 05 : 10 : 3,2

*l.* *s.* *d.* *q.*

Anſwer 2 : 05 : 10 : 3,2

## *Having the Diameters and Depth of a Conical Tun; To find the Depth or Heighth of the whole Cone, of which ſuch a Tun is a Fruſtum.*

### *EXAMPLE.*

Let ABCD be the Fruſtum of a Cone, and let the Diameter at the Baſe AB be 40 Inches; that at the Top CD=35. and let the Depth OP be 27: I demand the Heighth of a Cone, *viz.* ON, the Diſtance of the Vertex N, from the Center of the Baſe O.

*RULE.*

### RULE.

Say, as Half the Difference of the Diameters DC, and AB, *i. e.* A *d* or B *c* is to the Depth of the Fruſtum *c d* or D *c*: So is Half the Diameter of the Baſe AB, *i. e.* AO=BO, to the Heighth of the whole Cone NO.

| | *in.* |
|---|---|
| Greateſt Diameter | 40 |
| Leaſt Diameter | 35 |
| Difference | 5 |
| Half the Difference | 2,5 |

Half the Baſe AB is 20; then ſay,

A *d* : C *d* : : AO : NO.
As 2,5 : 27 : : 20 : 216.
20

2,5) 540,0 (216, anſwer, *i. e.* NO the Heighth re-(quir'd.
045(0
1(0

Here you may note, that the Triangles A, N, O, and A, C, *d*, are Similar; and therefore the Proportion preceding is grounded upon *Prop.* 4th of *Euclid* 6. where 'tis demonſtrated, that the Sides of Equiangular Triangles are Proportional.

## *Having the Heighth of a Cone, and the Diameter of its Baſe, to find the Diameter at any Depth.*

The Solution of this *Problem* depends upon the ſame Geometrical Principle with the laſt; and conſequently may have the ſame Figure to inform the Mind.

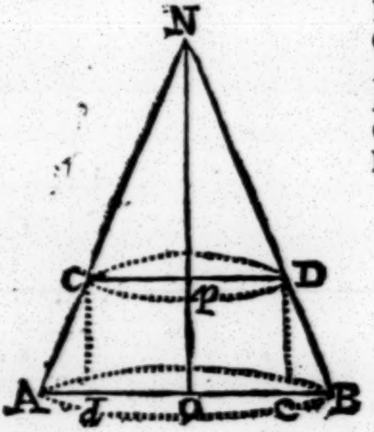
### EXAMPLE.

Let the Heighth be NO=216 Inches, and the Diameter AB=40; to find the Diameter CD at 27 Inches from the Baſe AB.

1. Take 27 = $cd$ = $po$, out of NO = 216, and there will remain 189, the Side N$p$ of the right-angled Triangle NC$p$.

2. Say, as the Side NO = 216, is to half the Diameter AO = 20, ſo is N$p$ = 189, to C$p$ = 17,5, which is half the Diameter CD; which double gives 35 = CD, the Diameter requir'd.

By the laſt Example you may find the Diameter at every 5 or 10 Inches Depth of any Conical Tun; and this will be of great Uſe in the Practice of Gauging



NO = 216
CD = 35
AB = 40
A$d$ = B$c$ = 2,5
C$d$ = D$c$ = $po$ = 27
N$p$ = 189.

## *Having the Diameter of a Circle to find its Circumference.*

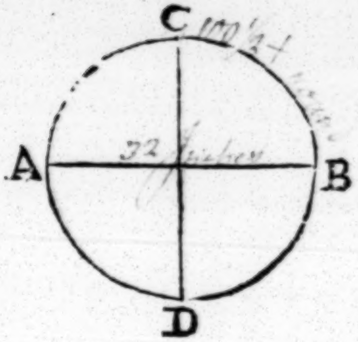
### RULE.

Say, as 1 is to 3,1416, or as 113 to 355, or as 7 is to 22; ſo is the Diameter to the Circumference.

### *EXAMPLE.*

Let the Diameter AB of the Circle ABCD, be 32 Inches, what's the Circumference?

1. As



1. As 1 : 3,1416 : : 32
32
62832
94248
*Facit* 100,5312, by the firſt Way.

2. As 113 : 355 : : 32
32
710
1065
113) 11360,0000 (100,5309, by the ſecond Way.
00060510(3
006311(8
0001(0

3. As 7 : 22 : : 32
22
64
64
7) 704,0000 (100,5714, by the third Way.
004513(2

This laſt Way I would recommend, being eaſy to remember, yet not ſo exact as either of the other, but near enough for indifferent Uſes.

## *Having the Circumference of any Circle to find the Diameter.*

Say, as 3,1416 is to 1, or as 355 is to 113, or as 22 is to 7, ſo is the Circumference of any Circle to its Diameter.

*EXAMPLE.*

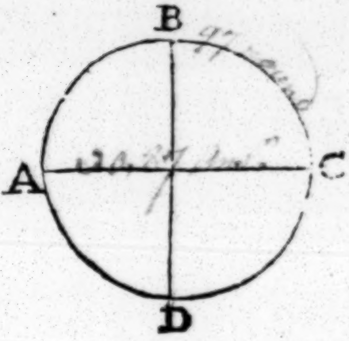
Let the Circumference be 97 Inches, what's the Diameter?

1. As 3,1416 : 1 : : 97
3,1416) 97,0000000 (30,875 Diam. requir'd, firſt Way.
02752028(0
275270(0
2388(0
18(1
(3

2. As 355 : 113 : : 97
97
791
1017
355) 10961,000 (30,876 Diam. the ſecond Way.
031105(0
3171(2
22(0

3. As 22 : 7 · : 97
7
22) 679,000 (30,863 Diam. the 3d Way.
01948(4
110(1

To



## *To find the Duty of any Number of Buſhels of Malt, ſuppoſing 'twas given in the Groſs.*

### *RULE.*

Multiply the Number of Groſs Buſhels by ,02; and the Product will be the Pounds or Decimal Parts of a Pound Sterling; which is the Duty thereof.

### *EXAMPLE.*

What's the Duty of 437,5 Buſhels of Malt Groſs?

,02

*l* *s.* *d.*

Anſwer 8,550, *i. e.* 8 : 11 : 00.

Having the Duty of any Number of Buſhels Neat, and the Groſs alſo, you may find the former Multiplicator by asking the Queſtion following; *viz.*

What Number is that by which if you multiply the Groſs Product ſhall be the Duty?

By this you know, that if you divide the Duty of the Neat in Pounds or Parts by the Groſs, the Quotient will be ,02 a common Multiplicator in this Caſe.

*EX-*

### EXAMPLE.

Let the Groſs be 20 Buſhels, the Neat 16, and the Duty 8 Shillings; that is, ,4 of a Pound Sterling, which divide by 20, the Groſs 20) ,40 (,02 is the Multiplicator
(0(0

## The Uſe of the *Sliding Rule*.

The Ingenious Mr. *Everard* has ſufficiently handled this Subject, therefore I might have ſaved my ſelf that Trouble; but becauſe 'twill be expected, and the Book more uſeful, I ſhall ſay ſomething to that Purpoſe, and begin firſt with

### *NUMERATION upon the Lines.*

Numeration by theſe Lines may be underſtood by this one Thought, *viz*. That what Denomination ſoever the 1 at the Beginning of the Line is, that in the Middle will be ten Times, and that at the End will repreſent a hundred Times ſo many; which, if underſtood, 'tis not hard to know what the intermediate Figures and Parts are.

### EXAMPLE.

To find the Place of 25. you may call the 1, at the Beginning of the Lines but 1, then will that in the Middle be 10, and the 2 which ſtands upon the ſecond *Radius*, will be 20; then count five of the grand Diviſions; where ſtop, for that is the Place which repreſents 25. Where note, that if you had eſteemed the 1 at the Beginning of the Line but 1, that is, one Tenth, the Place which now repreſents 25. would then ſignify but 2,5; alſo, if you had eſteemed it as 10, then would the Place of 25. be 250; if 100, then 2500; if but ,01, then but ,25, &c.

### EXAMPLE II.

Find upon the Line A or B, the Place of 3652.

Firſt, eſteem the 1, at the Beginning of either Line to be 100, then will that on the Middle be 1000, and the 3, which is on the ſecond *Radius* is 3000; from which count 6 of the grand Diviſions and a Half (towards 4000) and then you'll come to the Place of 3650. Now you muſt imagine the 2 to be a little beyond that half Diviſion; (for in this and the like Examples, where we are to find 4 Places, that which is Units muſt be taken by Eſtimation) ſo have you the Place of 3652.

### EXAMPLE III.

Find upon the ſame Lines the Place of 15737.

Firſt eſteem the 1 at the Beginning of the Line to be 1000, then will that in the Middle be 10000, and 5 of the grand Diviſions will be 5000, *i. e.* 15000; then is 3 and Half of the intermediate Diviſions the Place of 700, *i. e.* 15700: And as for the Place of 37, you muſt by Eſtimation take it a little farther forward. So will you come to the Places 15737. But here you are to note that you can't exactly find the Place of any Number which exceeds 4 Places; and in many Caſes you will not be ſure of the 4th Place. However, what has been ſaid is ſufficient to explain Numeration.

## *MULTIPLICATION by the Lines.*

This Rule (whether to be perform'd Arithmetically or Inſtrumentally) depends upon *Euclid's Elements*, *Prop.* 1. *Lib.* 2; where 'tis demonſtrated, that if two Lines be propos'd, whereof one is divided into diverſe Parts, the Rectangle contained under thoſe two Lines, is equal to the Rectangles contained under the Line which is not divided, and the Parts of the Line divided.

The Proportion is, as 1 is to the Multiplier; ſo is the Multiplicand to the Product.

### *EXAMLPE.*

Let it be requir'd to multiply 26 by 9.

### *RULE.*

Set 1 (upon the Line B.) to 9 upon A; then againſt 26 upon B, is 234, the Product ſought upon A. Where note, that when the Product conſiſts of three Places, the Rule will inform you what two Figures ſhould poſſeſs the Places of Tens and Hundreds, but what ſhould poſſeſs the Place of Units will ſometimes be doubtful; therefore, that they may be always certain what that Figure ſhould be, you may in your Mind multiply the unit Figure of the Multiplier by that of the Multiplicand, as above. I multiply 6 by 9, and the Product is 54; by which I know the 4 will poſſeſs the Units Place of the Product.

### *EXAMPLE* II.

Let it be required to find the Product of 72 in 26.

### *RULE.*

Set 1 upon B. to 26 upon A; then againſt 72 on B is 1872 upon A; and ſuch is the Rectangle of 26 in 72.

It ſometimes ſeems doubtful how many Figures ſhould be in the Product; to remedy which take the Rule following.

### *RULE.*

Obſerve how many Figures above Units are in the Multiplicand, and how many in the Multiplier; th[illegible]r

[illegible]ery

every one as there is in both, (above Units) reckon one in the Product; and for the Units in both, reckon one more. And laſtly, if the Multiplicand is found againſt the Product in the ſame *Radius* on which the Multiplier was, you'll have no more Places than you had by your former Account. But if it falleth beyond that *Radius*, you muſt account (for that) one Place more; as in the laſt Example, for the 2, I reckon 1, and for the 7, 1, *i. e.* 2. then the 2 in 72, and 6 in 26, are both in the Place of Units, for which I reckon one more, *i e.* 3; then when my Rule is ſet, *i. e.* when I ſet 1 on B, to 26 on the firſt *Radius* of the Line A, I find 72 upon B will not fall on the ſame *Radius*, on which 26 is, but the next beyond; for which I reckon another Place in the Product, which add to the 3 as before found, and the Sum, *viz.* 4, will inform me, that the Product will conſiſt of 4 Places, *viz.* 1872, as above.

## *DIVISION by the Lines.*

In Diviſion, the Quotient contains Unity as often as the Dividend does the Diviſor: It follows then from the 5th Definition to the 5th Book of *Euclid's Elements*, that the Quotient is in Proportion to 1, as the Dividend is to the Diviſor; from whence then we may alſo deduce the Proportion following, *viz.* as the Diviſor is to 1, ſo is the Dividend to the Quotient.

### *EXAMPLE.*

Divide 336 by 12.

### *RULE.*

Set 12 upon B, to 1 upon A; then againſt 336 upon B, you'll find 28 upon A; which is the Quotient required.

You'll know how many Figures ſhould be in the Quotient, by ſetting the Diviſor orderly under the Dividend, *&c.*

### *EXAMPLE.*

Divide 48627 by 96.

### *RULE.*

Set 96 upon B, to 1 upon A; then againſt 48627 on B, you have 506,53 the Quotient required upon A. So

$$\text{if}\left\{\begin{array}{l}26\\48\\96\end{array}\right\}\begin{array}{l}\text{were given to be divi-}\\\text{ded by 6, you'll find}\\\text{the Quotients to be}\end{array}\left\{\begin{array}{l}4.33,\ \&c.\\8.\\16.\end{array}\right\}$$

## *REDUCTION of Decimals by the Rule.*

Let this vulgar Fraction, *viz.* $\frac{3}{4}$, be given to be reduced unto a Decimal.

### *RULE.*

As the Denominator 4, is to its Numerator 3, ſo is 1,00 the decimal Denominator to ,75 the decimal Numerator: Which is the Decimal of $\frac{3}{4}$: *Where note*, that every Decimal will be as little in Reſpect of 1, as the Numerator is in Reſpect of the Denominator of its vulgar Fraction. Set 4 upon B, to 1 upon A, then againſt 3 upon B, is ,75 on A; and that is the Decimal of $\frac{3}{4}$: For as 3 the Numerator is to 4 its Denominator ſo is ,75 a decimal Numerator to 100, its Denominator, &c.

### *EXAMPLE* II.

What's the Decimal of 18 Shillings making a Pound Sterling the Integer?

The vulgar Fraction is $\frac{18}{20}$; therefore ſet 20 upon B, to 1 upon the Line A, then againſt 18 on B, is ,9 on A; which is the Decimal required.

### *EXAMPLE* III.

What decimal Part of a Barrel is 12 Gallons? The vulgar Fraction is $\frac{12}{34}$; therefore ſet 34 upon B, to 1 upon on A; then againſt 12 on B, is ,35, *&c.* on A; which is the Decimal of 12 Gallons required.

## *To find the Value of the Decimal of a Pound Sterling.*

### EXAMPLE I.

What is the Value of ,27 of a Pound Sterling?

1. Set 1 upon B, to 20 upon A; then againſt ,27 on B, is 5,4 on A, *i. e.* 5 Shillings, 4 Tenths.

2. Set 1 upon B to 12 upon A; and then againſt 4 on B, you'll find 4,8 on A, *i. e.* 4 Pence, and 8 Tenths of a Penny.

3. Set 1 upon B to 4 upon A; then againſt 8 on B is 3,2 on A, *i. e.* 3 Farthings, and ,2 of a Farthing. And thus by the Rule you find, that the Value of ,27 of a Pound Sterling, is 5 Shillings 4 Pence 3 Farthings, and ,2 of another.

### EXAMPLE II.

What is the Value of ,62 of a Barrel of Beer?

1. Set 1 upon B, to ,62 upon A; then againſt 4 upon B, is 248 on A, from which (according to Multiplication) cut off two Places, and then 'tis 2,48, *i. e.* 2 Firkins, and ,48 Parts.

2. Set 1 upon B, to ,48 on A; then againſt 8,5 on B, is 4080 on A, from which cut off three Places, and then 'twill be 4,080, *i. e.* 4 Gallons and ,080 of a Gallon.

*F. Gall.*

So the Value of ,62 of a Barrel of Beer is 2 : 4 : 08. And according to this Method may the Value of any Decimal be found, either in Weight, Meaſure, Money, or Time, to a very inconſiderable Matter.

## *The Rule of* THREE *by the Lines.*

The Rule of Three by the Lines muſt be perform'd by the Method following: *viz.*

Set the Firſt Number on B, to the ſecond on A, then againſt the third Number on the Line B, you'll have the fourth required on A.

### EXAMPLE I.

If the Duty of 16 Buſhels of Malt be 8 Shillings; what's the Duty of 47 Buſhels at that Rate?

### OPERATION.

Set 16 upon B, to 8 upon A; then againſt 47 on B, you'll find 23,5 on A, *i. e.* 23 Shillings 6 Pence, the Anſwer.

### EXAMPLE II.

If the Proportion to find the Circumference of a Circle be as 113. to 355; I demand the Circumference of a Circle, whoſe Diameter is 26.

### OPERATION.

Set 113 upon B, to 355 upon A; then againſt any Diameter on B, you'll have its Circumference on A; So againſt 26 on B, you have 81,68 on A, the Circumference required.

### EXAMPLE III.

Having the Circumference of a Circle to find the Diameter, the Proportion is as 355 to 113; ſo is any Circumference to its Diameter.

Let the Circumference be 96, what's the Diameter?

### OPERATION.

Set 355 upon B, to 113 on A; then againſt any Circumference upon the Line B, you'll have its Diameter on A; ſo againſt 96 on B, you'll find 30,5 on A, the Diameter required.

### EXAMPLE IV.

The Proportion between Beer and Wine, is as 9 to 11.

Suppoſe by the following Rules, that I found a Veſſel to contain 42 Gallons of Beer; I demand how many Gallons of Wine the ſame will hold.

### OPERATION.

Set 9 upon B, to 11 upon A; then againſt 42 on B, you'll find 51,33, *i. e.* 51 Gallons, ,33 Parts of Wine on A, the Anſwer.

## *To find the Area of a Circle in Beer and Wine Gallons, and Malt Buſhels.*

### RULE.

Say, As the Gauge Point (proper to that Denomination in which you'll find the Area) on the Line D, is to 1 on C; ſo is any Diameter on D, to its Area on C.

### EXAMPLE.

There is a Circle whoſe Diameter is 56 Inches; what is the Area in Beer and Wine Gallons, and Malt Buſhels?

For Beer, ſet the Gauge Point, *viz.* 18,95, marked with AG, on the Line D, to 1 on C; then againſt 56 on D, you'll find 8,73 on C, and as the Rule is now ſet, it may ſerve as a Table of Circles Areas; for againſt any Diameter on D, you'll find its Area on C.

For Wine, ſet the Gauge Point, *viz.* 17,15 (mark'd with WG) on the Line D, to 1 on C; then againſt 56 on D, you'll find 10,666 on C, the Area requir'd.

**For Malt,** ſet the Gauge Point, *viz.* 52,32 (mark'd with MB) on D, to 1 on C; then againſt 56 on D, you'll find 1,145, on C, the Area requir'd. Here note, that if you'd have $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, *&c.* of the Area of a Circle, you muſt ſet the Gauge Point, to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, *&c.* of 1, and then againſt the Diameter you'll find the like Part of its Area.

## *To extract the* Square-Root *by the Rule.*

This Rule is grounded upon the 4th *Prop.* of the 2d Book of *Euclid's Elements*, but may be performed thus: Set thoſe Lines, which are noted with the Letters C and D, even; *i. e.* ſet 10 at the End of the Line D, to 10 which is at the End of the Line C; then theſe two Lines will be as a Table of Squares and their Roots; for againſt any Square on the Line C, you'll have its Root on D.

When note, That when the Figures in the Number given are even, *i. e.* when they conſiſt of 2, 4, 6, or 8 Figures, (being Integers,) look the ſame in the ſecond *Radius* of the Line C, and againſt it you'll have the Square Root thereof on the Line D.

### *EXAMPLE.*

Let 64 be given to have its Square Root extracted.

### *RULE.*

Having ſet the Lines C and D even, find 64 in the ſecond *Radius* of the Line C, and againſt it upon D, you'll find 8, the Square Root requir'd.

| | | | |
|---|---|---|---|
| In like manner, as the Rule now ſtands, you'll find that | 4<br>5<br>6<br>7 | are the Square Roots of | 16<br>25<br>36<br>49 |

When the Integers in the Number given are odd, as conſiſting of, 1, 3, 5, 7, or 9 Places, you muſt find it in the firſt *Radius* of the Line C, and againſt it you'll have the Root requir'd on the Line D.

*EXAMPLE.*

Let 144 be given to have its Square Root extracted.

*OPERATION.*

Find 144 upon the firſt *Radius* of the Line C, and againſt it on the Line D, you'll have 12, the Root required.

$$\text{So will} \left\{\begin{array}{r} 14{,}2 \\ 2{,}5 \\ 10. \end{array}\right\} \begin{array}{l} \text{be the Square Roots} \\ \text{of} \end{array} \left\{\begin{array}{r} 201{,}64 \\ 6{,}25 \\ 100, \end{array}\right.$$

To know of how many Places the Root will conſiſt, you may (before you handle the Sliding Rule) point the Square given, as is before directed.

## The Uſe of the *Square Root* by the Sliding Rule.

### PROB. I.

*To find a mean Proportional between any two given Numbers.*

*RULE.*

Set either of the two given Numbers upon C, to the ſame on D; then againſt the other Number given on C, you'll find the Mean requir'd on D.

EX-

### EXAMPLE.

Find a mean Proportional between 64 and 256.

### OPERATION.

Set 64 upon the Line C, to 64 on D; and then againſt 256 on C, you'll have 128, the Mean required on D.

Ead,

## PROB. II.

### *Given the Conjugate and Tranſverſe Diameters of an Oval, to find the Diameter of a Circle which ſhall be equal to the Oval in Area.*

### EXAMPLE.

Let the conjugate Diameter of the Oval be 52,5, and the Tranſverſe 210.

### OPERATION.

Set 52,5 upon the Line C, to 52,5 upon the Line D; then againſt 210 upon C, you'll have 105 on D, which is the Diameter requir'd.

Ea° 104,7

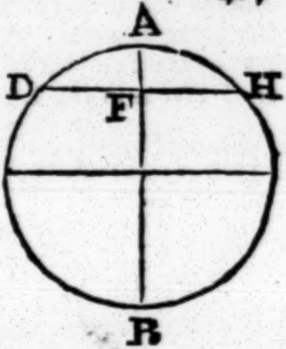
## PROB. III.

### *Given* AB *the Diameter of a Circle, and* AF, *the verſed Sine of its Segment, to find* DH *the Chord Line. Let* AB *be* 46 *Inches, and* AF, 8; *(*DF *is a mean Proportional between* AF *and* BF *by the* 13*th of* Euclid *the 6th.)*

### OPERATION.

Set 8 upon C, to 8 upon D; then againſt 38 = BF on

C, you'll have 17.43=DF, on D; which doubled, gives DH the Chord Line requir'd, which is 34,86 Inches.



3
86
100 | 258 | 2 [illegible]
200
.58

## To extract the *Cube Root* by the Lines E and D.

### GENERAL RULE.

Place the two Lines, *viz.* E and D, even; *i. e.* place 1 at the Beginning of E, to 1 at the Beginning of D, then against any Cube upon the Line E, you'll have its Root upon the Line D.

### EXAMPLE I.

What's the Cube Root of 1728? Set the Rule even as above directed, then find 1728 upon the Line E, and against that on the Line D, you'll find 12, the Cube Root required.

### EXAMPLE II.

What's the Cube Root of 6592? Set the Rule, and against 6592, upon the Line E, you'll find 18,75 upon the Line D, which is the Cube Root required.

Here note, That when the Integers in the given Cube conſiſt of 1, 4, or 7 Places, you muſt find it in the firſt *Radius* of the Line E; if of 2, 5, or 8, in the ſecond *Radius* of the Line E: But if of 3, 6, or 9 Places, then in the third *Radius* of the ſame Line. And here alſo you muſt point your Cube, as by the Pen you were before taught, to know the Number of Places in the Root.

## The Uſe of the *Cube Root* by the Sliding Rule.

### PROB. I.

*There is a Bullet whoſe Diameter is* 4 *Inches, and Weight* 9 *Pounds; what is the Weight of another Bullet, whoſe Diameter is* 16 *Inches?*

#### *RULE.*

Set 4, the Diameter given, on the Line D; to 9*l.* the Weight of that Bullet upon the Line E; then againſt 16 upon the Line D, you'll find 576 upon the Line E; and that is the Weight of the Bullet required.

### PROB. II.

*There is a Bullet whoſe Diameter is* 8 *Inches, and Weight* 72 *Pounds; I demand the Weight of another Bullet (of the ſame Metal) whoſe Diameter is* 4 *Inches.*

#### *RULE.*

Set 8 (the given Diameter) upon D, to 72 the Weight of its Bullet on E; then againſt 4 upon D, you'll find 9 upon E, the Weight required. And, as the Rule now ſtands, you may find the Weight of all Bullets of that Metal; for againſt the Diameters upon D, you'll have the Weight on E.

Thus, if the Diameter of a Bullet be … Inches, the Weight will be … *l.*

| Diameter (Inches) | Weight (*l.*) |
|---|---|
| 3 | 3,79 |
| 5 | 17,57 |
| 6 | 30,37 |
| 7 | 48,23 |

## PROB. III.

*Having the Diagonal Line of a Cask taken as the middle Fruſtum of a Spheroid in Inches, to find the Content thereof in Beer Gallons.*

### *RULE.*

Set 22,93 upon the Line D, to 27 upon the Line E; then againſt the Diagonal of any ſpheroidal Cask upon D, you'll have the Content thereof upon the Line E, in Beer Gallons.

### *EXAMPLE.*

Let the Diagonal of a Cask be 20 Inches; I demand the Content thereof in Beer Gallons.

### *OPERATION.*

Set 22,93 upon D, to 27 upon the Line E, then againſt 20 (the given Diagonal) upon D, you'll find 17,91 upon E, and that is the Content required. And as the Rule now ſtands, you'll find, that if the Diagonal Line in Inches of any ſuch Veſſel

| be | The Content thereof in Beer Gallons will be |
|---|---|
| 12. | 3,86 |
| 22,93 | 27. |
| 48,59 | 257. |

The foregoing Examples depend on the ſame Principle, on which I ſignified the former Problems in the Uſe of the Cube Root Arithmetically did; *i. e.* that like Solids are in triplicate Proportion of their Homologous Sides; and on the ſame depend thoſe which follow.

## PROB. IV.

### *Having the Diagonal of a Veſſel in Inches, to find the Content thereof in Wine Gallons.*

### *RULE.*

Set 15,9 upon D, to 11 upon E, and then againſt any Diagonal Line in Inches upon D, you'll have the Content of the Veſſel on E, in Wine Gallons.

### *EXAMPLE* I.

Let the Diagonal of a Cask in Inches be 50,49; I demand the Content in Wine Gallons.

### *OPERATION.*

Set 15,9 upon D, to 11 upon E; then againſt 50,49 upon D, you'll have 352 upon E; and ſuch is the Content required in Wine Gallons.

### *EXAMPLE* II.

Suppoſe the given Diagonal was 9,01; I demand the Content in Wine Gallons.

### *OPERATION.*

Set 15,9 upon D, to 11 upon the firſt *Radius* of the Line E, then againſt 9,01 (the given Diagonal) on D, you'll find 2, upon the Line E, for the Content of the Veſſel in Wine Gallons.

The two laſt Problems will be of great Uſe where a four Foot Rule is not at Hand, to find the Content of

Casks (if upon the Sliding Rule you have the Lines E and D projected) only by knowing the Length of the Diagonal Line in Inches. But inſtead of theſe Lines, there's now commonly inſerted a Line of Numbers and Segments for a ſtanding Cask, of which I ſhall ſpeak in its proper Place; but theſe are not (in my Opinion) ſo neceſſary as the other.

## PROB. V.

*There is a Conical Tun, whoſe Diameter at the Top is 20 Inches, that of the Bottom 26 Inches, and the Depth 30: By which the Content (in Ale Gallons) appears to be 44,45: I demand the Diameter at the Top, the Diameter at the Bottom, and the Depth of another Conical Tun, whoſe Content is 88,9 Gallons.*

### *OPERATION.*

1. To find the Diameter at the Top.

Set 44,45 upon E, to 20 upon the Line D; then againſt 88,9 upon E, you'll find 25,2, the Top Diameter required.

2. To find the Bottom Diameter.

Set 44,45 upon E, to 26 upon the Line D; then againſt 8,89 upon E, you'll find 32,75, the Bottom Diameter required on D.

For the Depth of the Conical Tun, whoſe Content is 88,9.

Set 44,45 upon the Line E, to 30 upon the Line D, then againſt 88,9 on E, you'll find 37,79, the Depth required upon D.

## PROB. VI.

*There is a Cask whoſe Bung Diameter is* 20 *Inches, its Head Diameter is* 14, *and the Length* 24; *by which the Content appears to be* 22,19 *Beer Gallons; I demand the Bung, Head, and Length of another ſuch Veſſel, whoſe Content is* 40 *Gallons.*

### *OPERATION.*

1. To find the Bung Diameter.

Set 22,19 upon E, to 20 upon D; then againſt 40 upon E, is 24,34, the Bung Diameter required.

2. To find the Head-Diameter.

Set 22,19 upon E, to 14 upon D; then againſt 40 upon E, you'll find 17,03 upon D, for the Head Diameter required.

3. To find the Length.

Set 22,19 upon E, to 24 upon D; then againſt 40 upon E, is 29,2, the Length required on D.

---

## The Uſe of the *Rule* in Meaſuring Timber, Stone, Boards, *&c.*

## PROB. I.

*The Diameter of a Circle given to find the Circumference.*

### *RULE.*

SET 1 upon A, to 3,14, *&c.* on B, (which Point is marked with C for Circumference,) and then againſt any Diameter on A, you'll find its Circumference on B.

EXAMPLE.

Let the Diameter of a Circle be 40 Inches; what's the Circumference?

Set 1 on the Line A, to the Point c. on the Line B; then againſt 40, on the Line A, you'll find 125,66 on the Line B, and ſuch is the Circumference required.

## PROB. II.

### *The Circumference given to find the Diameter.*

RULE.

Set the Point c. 3,141, *&c.* on B, to 1 on A. then againſt any Circumference on B, you'll find its Diameter on A.

EXAMPLE.

Let the Circumference of a Circle be 125,66 Inches; what's the Diameter?

Set the Point c. upon the Line B, to 1 on the Line A; then againſt 125,66 on the Line B, you'll find 40, the Diameter required on A.

## PROB. III.

### *The Diameter of a Circle given, to find the Side of a Square which ſhall be equal in Area thereto.*

RULE.

Set the Point (Se) *i. e.* 8862 on the Line B, to 1 on A; then againſt the Diameter of any Circle on A, you'll find the Side of an equal Square on B.

### *EXAMPLE.*

Let the Diameter of a Circle be 40 Inches; what's the Side of an equal Square?

Set S. e. (ſignifying Square equal,) *i. e.* ,8862 on B, to 1 on A; then againſt 40 on A, is 35,44 on B. and ſo much is the Side of a Square, whoſe Area will be equal to that Circle, of which 40 is the Diameter.

## PROB. IV.

### *The Diameter given, to find the Side of the greateſt Square that can be drawn within the Circle to which that is a Diameter.*

### *RULE.*

Set ,707106 mark'd with S. i. (ſignifying Square inſcrib'd) on B, to 1 on A; then againſt any Diameter on A, you'll find the Side of the inſcrib'd Square on B.

### *EXAMPLE.*

Let the Diameter of a Circle be 40 Inches; what's the Side of the inſcrib'd Square?

Set the Point Si=,707, *&c.* on B, to 1 on A; then againſt 40 on A is 28,284, the Side of the Square required.

## PROB. V.

### *The Diameter of a Circle given to find the Area thereof.*

Set 1 upon D, to ,78539 (mark'd with O, d.) on C, then againſt any Diameter on D, you'll find the Area required on C.

*EXAMPLE.*

Let the Diameter of a Circle be 30 Inches; what's the Area?

Set 1 upon D, to ,78539 on C, then againſt 30 on D, you'll find 706,85 on C, for the Area required.

## PROB. VI.

### *The Circumference of a Circle given to find the Area.*

*RULE.*

Set 1 upon D, to ,079578 ( mark'd with ⊖ *c.* ) on C, then againſt any Circumference on D, you'll find the Area required on C.

*EXAMPLE.*

Let the Circumference of a Circle be 60 Inches; what's the Area?

Set 1 upon D, to ,079578=⊖. c. on C, then againſt 60 on the Line D, is 286,48 on C, the Area required.

## PROB. VII.

### *The Breadth of a Board given in Inches, and the Length in Feet, to find the Content in Feet.*

*RULE.*

Set 12 on B, to the Breadth of the Board on A; then againſt the Length of the Board on B, you'll find the Content on A.

*EXAMPLE.*

Let the Breadth of a Board be 20 Inches, and the Length 18 Foot; what is the Content in Feet?

Set 12 on B, to 20 on A; then againſt 18 on B, is 30 on A, and that is the Area of the Board required.

## PROB. VIII.

### *The Length and Breadth of a Board given in Inches, to find the Content in Feet.*

*RULE.*

Set 144 on the Line B, to the Breadth of the Board on A; then againſt the Length on B, you'll find the Content on A.

*EXAMPLE.*

Let the Breadth of a Board be 20 Inches, and the Length 168; what's the Content in Feet?

Set 144 on B, to 20 on A; then againſt 168, on B, is 23,33 the Area required in Feet on A.

## PROB. IX.

### *The Breadth of a Board given in Inches, to find how much in Length thereof will go to make one Foot Square.*

*RULE.*

Set the Breadth given in Inches on B, to 1 on A; then againſt 144 on B, you'll find how much of the Board muſt be accounted one Foot Square.

*EXAMPLE.*

Let a Board be 16 Inches broad, how much in Length will go to make one Foot Square?

Set 16 upon B, to 1 on A; then againſt 144 on B, is 9

9 on A, and ſo much of that Board will go to make one Superficial Foot.

## Of *Timber* or *Stone.*

### PROB. X.

*Let a Piece of Square Timber or Stone be given to be meaſured, whoſe Length is* 14 *Foot, and Side* 17 *Inches; what's the Content in Solid Feet?*

Set 12 upon D, to the Length; *viz.* 14 Foot on C; then againſt the Side of the Square, *viz.* 17 Inches on D, you'll find 28,097 on C, which is the Content required.

But if the Dimenſions are given both in Foot Meaſure, then the Proportion is, *As* 1 upon D, is to the Length on C; ſo is the Side on D, to the Content on C.

### PROB. XI.

*Let a Piece of Timber be given, whoſe Length is* 22 *Foot, Breadth* 18 *Inches, and Depth* 13; *what's the Content in Solid Feet?*

The common Way of meaſuring unequal-ſided Timber, is to add the Breadth and Depth together, then Half that Sum is taken for the Square of the Piece: But this is an Errour too commonly uſed. However to proceed in our Buſineſs, you muſt find a mean Proportional betwixt the Depth and Breadth, thus ſet 18 on C, to 18 on D, then againſt 13 on C, you'll find 15,29 on D, and that is the Side of a Square Piece of Timber; which being of the ſame Length with that given, will be equal to it; therefore ſet 12 on D, to 22 Foot, the Length of the Piece on C; then againſt 15,29 on D, you'll find 35,71 on the Line C; and ſo much is the Content of the Piece required.

## PROB. XII.

*The Side of any Square Piece of Timber given in Inches, to find how many Inches in Length of the ſame Piece will go to make one Foot Solid.*

### *RULE.*

Set the Side of the Square given on D to 1, on C; then againſt 41,5 on D, you'll find what's requir'd on C.

### *EXAMPLE.*

Let the Side be 15 Inches; how much in Length of the Piece will make one Foot Solid?

Set 15 on D, to 1 on C; then againſt 41,5 on D, is 7,68 on C; and ſo many Inches in Length will make one Foot Solid of that Piece.

## Of Round *Timber*

## PROB. XIII.

*Let the Circumference of a Round Tree be* 40 *Inches, and the Length* 16 *Foot; what's the Content in Feet?*

### *RULE.*

Always ſet 42,53 on D, to (the given Length in Feet as in this Caſe) ,16 on C, then againſt the Circumference on D, *viz.* 40, you'll find 14,15 for the Content in Feet and Parts.

## PROB. XIV.

*Let the Circumference of a Round Tree be 92 Inches; I would know how much thereof in Length would make one Foot Solid.*

### *RULE.*

Set the Circumference, *viz.* 92 on D, to 12 upon C; then againſt 42,53 (a fix'd Number for this purpoſe) on D, you'll find the Length of a Foot required upon C, *viz.* 2,56. And the like may be done by any other Queſtion of the like Nature. But ſuch Perſons as deſire to ſee more of this, may peruſe Mr. *Everard's Stereometry*, the Third or Fourth Edition.

And having thus given Directions to a Perſon who deals in Boards, Timber, or Stone; 'twill next be convenient to add ſomething further about the Meaſuring of Wainſcot, Plaiſtering, and Paving; which are generally agreed for by the Yard.

And therefore, when you have taken your Dimenſions in Feet, and caſt them up, divide the Sum by 9, ſo will you have the Content in Square Yards.

In taking the Dimenſions (with a Line) you are to Girth over every Turn or Part of the Corniſh, and all Parts of the Wainſcot where the Mouldings ſwell much, from the Top of the Room to the Bottom, turning the Line over every ſuch Part with the Thumb of your Right-Hand.

But you muſt Girth the Room without turning the Line upon the Mouldings, as you do in taking the Heighth.

Where obſerve, that in meaſuring Window-Shutters and Doors, you muſt allow Meaſure and half Meaſure; that is, whatever you meaſure a Door or a Shutter at, it muſt be accounted Half as much more.

## *EXAMPLE.*

The Heighth of a Room is 14 Foot 9 Inches, and Compaſs 146 Foot 6 Inches.

In which there are two Doors, each 6 Foot 3 Inches Long, and 4 Foot Wide.

Alſo 4 Windows, each 2 Leaves for Shutters, which are 5 Foot 2 Inches High, and two Foot Broad.

What's the Content of ſuch a Room in Square Yards?

| | |
|---|---|
| Compaſs of the Room is | 146,5 |
| Its Heighth | 14,75 |
| | 7325 |
| | 10255 |
| | 5860 |
| | 1465 |
| **Content of the Room** | 2160,875 Square **Feet.** |

**Including Doors, Windows,** and Chimney.

| | | |
|---|---|---|
| **One Leaf of a Window-**Shutter is | 5,166 | **Feet Long.** |
| | 2. | **Wide.** |
| Content of one in Square Feet is | 10,332 | |
| Number of Shutters or Leaves | 8. | |
| Content of all in Square Feet | 82,656 | |

Length

| | *Foot.* | |
|---|---|---|
| Length of one Door is | 6,25 | |
| And Breadth | 4. | |
| Content in Square Feet | 25,00 | |
| Number of Doors | 2. | |
| Their Content | 50. | |
| To which add the Shutters | 82,656 | |
| Their Sum is | 132,656 | |
| Half their Sum is | 66,328 | for half Work. |
| To this add the Room | 2160,785 | |
| The Content is | 2359,769 | |

| | *F.* *In.* |
|---|---|
| The Heighth of a Chimney is | 4 : 6=4,5 |
| Its Breath is | 3 : 3=3,25 |
| | 14,625 |
| | 282,4155 |
| Content of the Chimney is | 14,625 |
| Which take from the Content above | 2359,769 |
| And there will remain the true Content of the Room | 2345,144 |

Which divide by 9, will give Square Yards.

```
9)  2345,144  (260,571 Yards.
    18
    ---
     54
     54
    ---
     051
      45
      ---
       54
       63
       ---
        14
         9
        ---
         5
```

Painters take their Dimenſions, and caſt up, as Joyners do, but are not to account for Work and Half, but to account (in ſome Caſes) the Room once, twice, or three Times done over, or ſo much *per* Yard, conſidering the Goodneſs of the Work; as white Painting is worth about 1 Shilling *per* Yard. Wall-Nut-Tree Painting is worth 1 *s.* 4 *d.* or 1 *s.* 6 *d.* *per* Yard, and ordinary Branch Painting is worth 12 *d.* 14 *d.* or 16 *d.* *per* Yard, Japan Work, Plain, either Black or White, is worth 3 *s.* 6 *d.* or 4 *s.* *per* Yard, Gates and Out-doors are worth Painting 8 *d.* or 9 *d.* *per* Yard.

## By the *Sliding Rule.*

Say, as 9 upon B, is to the Breadth upon A; ſo is the Length upon B, to the Content in Square Yards upon A.

### EXAMPLE.

Let the Length be 76,5 Feet, and Breadth 17,5 Feet; what's the Content in-Square Yards?

Set 9 upon B, to 17,5 upon A; then againſt 76,5 on B, is 148,75 Square Yards upon A.

And the ſame Method ſerves either in Painting, Plaiſtering, Joynery, or Paving.

But 'tis common among Plaiſterers to make no Allowance for Doors, Windows, or Chimnies.

## *For Flooring, Partitioning, Rooſing*, &c.

Partitioning, Flooring, Roofing, and Tyling are meaſured by the Square of Ten, each Square being 100 Feet.

### EXAMPLE I.

Let the Length of a Floor be 76,2 Feet, and its Breadth 22,6 Feet, to find its Content in Squares.

```
100 : 76,2 : : 22,6
        22,6
      ------
        4572
       1524
      1524
      ------
Anſwer  17,2212  Squares.
```

By the Sliding Rule, ſet 100 on B, to 76,2 on A; then againſt 22,6 on B, you'll 17,22 on A.

### EXAMPLE II.

Let the Length of a Building be 125 Feet, and Breadth

49; I demand how many Squares of Roofing or Tyling it contains.

As 100 the Square Feet in a Square, is to 75,5, (twice the Length of the Rafter in Feet,) ſo is the Length 125, to 94,375 the Content in Squares, (*i. e.*)

$$\begin{array}{r} 100 : 75{,}5 :: 125. \\ 125. \\ \hline 3775 \\ 1510\phantom{0} \\ 755\phantom{00} \\ \hline 94{,}375 \end{array}$$

By the Sliding Rule, ſet 100 on B, to 125 on A; then againſt 75,5 on B, is 94,37 on A, as above.

Here obſerve, That the Breadth of the Floor, is to the Length of the Rafter, as 4 to 3, to which you muſt add the Vallies and Eves.

## To Meaſure Brick Work.

Brick Work is caſt up by the Rod Square, each containing 272,25 Square Feet.

Where you muſt remember, that all Brick Work muſt be reduced to the Standard Meaſure of one Brick and Half thick, which may moſt eaſily be done by the Help of the following Table; which goes as far as any Brick Wall is in Thickneſs at this Time.

### *EXAMPLE* I.

Let the Heigth of a Wall be 22,5 Feet, and Length 156,25; I demand the Content in Square Rods at 1 Brick and ½ Thick.

Look in the Table for the given thickneſs, *viz.* 1,5 and againſt it you'll find 272,25; then ſay

272,25 : 156,25 : : 22,5
22,5
78125
31250
31250
272,25) 3515,6250 (12,91 Anſwer.
793120(5
24860(7
036(7
(8

### EXAMPLE II.

Let ~~the Height of~~ a Wall be 2 Bricks Thick, 243 Feet Long, and 16,2 Feet High; what's the Content in Square Rods?

Look in the Table for 2,0, and againſt it you'll find 204,19 : Then ſay

204,19 : 243, : : 16,2
16,2
486
1458
243
204,19) 3936,6000 (19,27 Anſwer.
20419
189470
183771
56990
40838
161520
142933
18547

Suppoſe this Wall to have a Water Table 3 Bricks Thick, and 4 Foot High; what's the Content of the whole Wall?

Againſt 3 in the Table, you'll find 136,12; then ſay

```
136,12 : 243, : : 4
                  4
          ─────────
136,12)  972,0000  (7,14
         95284
         ─────
          19160
          13612
          ─────
           55480
           54448
           ─────
            1032
```

| | | |
|---|---|---|
| The upper Part of the Wall is | 19,27 | Square Rods. |
| The Water Table is | 7,14 | Square Rods. |
| The whole Content of the Wall | 26,41 | Square Rods. |

When you meaſure Brick Work, you muſt find the Content of each Wall ſeverally, and from it ſubſtract all the Doors and Windows.

## By the *Sliding Rule.*

Set 272,25 on B, to the Length of a Wall (of any Thickneſs) on A; then againſt the Breadth on B, you'll find the Content in Square Rods on A; if the Wall be juſt 1 ½ Brick Thick, but if more or leſs, 'twill require a ſecond Operation, thus: Set 3 on B, to the Number of half Bricks, the Wall is thick on A; then againſt the Content or fourth Number found by the firſt Operation,

tion, on B, you'll find the true Content of the Wall, *&c.* on A: Or, ſet the tabular Number found againſt the Thickneſs on B, to the Length of a Wall on A; then againſt the Breadth on B, you'll find the Content on A, at once.

| Bricks Thick. | Square Feet in a Rod. |
|---|---|
| 0,5 | 816,75 |
| 1,0 | 403,37 |
| 1,5 | 272,25 |
| 2,0 | 204,19 |
| 2,5 | 163,35 |
| 3,0 | 136,12 |
| 3,5 | 116,67 |
| 4,0 | 102,09 |
| 4,5 | 90,75 |
| 5, | 81,68 |
| 5,5 | 74,25 |
| 6, | 68,06 |
| 6,5 | 62,82 |
| 7, | 58,34 |
| 7,5 | 54,45 |
| 8, | 41,04 |
| 8,5 | 48,04 |
| 9, | 45,37 |
| 9,5 | 42,98 |
| 1,0 | 40,83 |

Tom
Rea Coley

*Note,* That about a Thouſand of Bricks, will do a Rod of Work on a two Brick Wall

The laying of which Number, will be about a Days Work for a Bricklayer and a Labourer.

Glaſiers meaſure their Work by the Foot Square, and is the ſame with that of meaſuring Boards; the Dimenſions of either may be taken in Feet and Inches, and the Content found by croſs Multiplication.

### EXAMPLE.

| | F. | In. | |
|---|---|---|---|
| Let the Breadth of a Pane of Glaſs be | 4 | 3 | |
| And its Length | 6 | 6 | |
| | 24 | | |
| | 1 | 6 | |
| | 2 | 0 | |
| | 0 | 1 | 6 |
| Content in Square Feet, Inches, and Parts. | 27 | 7 | 6 |

Which is done thus.

1. Firſt multiply 4 Foot by 6 Foot, and the Product is 24 Foot; which ſet under F.

2. Then 3 Inches by 6 Foot, gives 18 Inches, or 1 Foot 6 Inches; which ſet down under the former, as you ſee.

3. Multiply 4 Foot by 6 Inches, the Product is 24 Inches, or 2 Foot.

4. Multiply 3 Inches by 6 Inches, and the Product is 18 Parts; which divide by 12, and the Quotient is 1 Inch, and 6 Parts remains, which ſet down.

*Laſtly*, Add all into one Sum, and you'll have the Anſwer, *viz.* 27 Foot, 7 Inches, and 6 Parts.

And after the ſame Manner, you may caſt up any Thing elſe, whoſe Dimenſions are given in Feet and Inches, *&c.*

In many Caſes, 'twill be neceſſary to take your Dimenſions in Feet, Inches, and Parts, *i. e.* to Halves and Quarters, (as in Glazing, *&c.*) in which for a Quarter of an Inch, ſet down 3 Parts.

For half an Inch, you muſt ſet down 6 Parts.

And for three Quarters of an Inch, 9 Parts, (ſuppoſing

12 Parts make an Inch as 12 Inches does a Foot,) here note, That

Feet multiplied by Feet, give Feet.
Feet by Inches, give Inches.
Feet by Parts, give Parts.
Inches by Inches, give Parts.
Inches by Parts, give Seconds.
Parts by Parts, give Thirds.

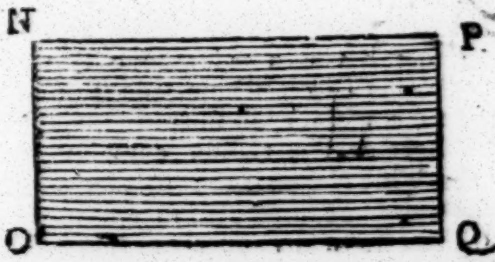
---

## *Gauging* perform'd Arithmetically and Inſtrumentally.

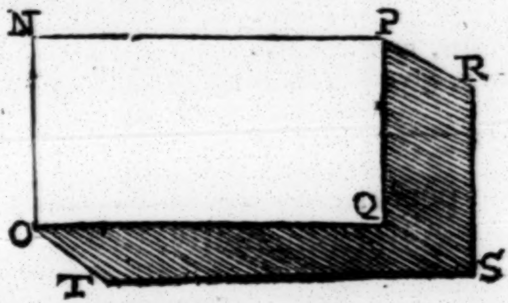
WHen the young Gauger has ſtored his Memory with the foregoing Notions, he may venture upon this Part; but firſt ought to know ſomething of the *Geneſis* of Quantity, *i. e.* That a Point begets a Line, as the Point A, moved from B to C, will leave the Trace, or deſcribe the Line BC,

A (.) B————.————C.

And a Line begets a Surface, if we conceive it to be mov'd through any Space, *parallel* to its ſelf, as we conceive the Line BC to be moved parallel to it ſelf, between the Lines NO and PQ, it will by that Motion, deſcribe the Surface NOPQ.



Laſtly. If we imagine this Surface to ſink Perpendicularly and Parallel to its ſelf, it will, by ſuch a Motion, deſcribe a Solid, as NOPQRST.



1. A Point is ſaid to be that which has no Parts.
2. A Line is that which has Length, but not Breadth.
3. A Surface is that which has both Length and Breadth.
4. A Solid admits of Length, Breadth and Thickneſs.

Having a clear Apprehenſion of what has been ſaid, we are to learn, that a Line is meaſured by a Line; as a Line 2 Foot long, may be meaſured by a Line that is one Inch long, and be found to contain 24 ſuch.

And that a Superficies, muſt be meaſured by a Superficies: So a Superficies that is one Foot Square, may be meaſured by another Superficies, that is but one Inch Square, and found to contain 144 ſuch little Squares.

Alſo, that a Solid muſt be meaſured by a Solid.

So a Cube that has each Side = one Foot, may be meaſured by another Cube, whoſe Side is one Inch, and will be found to contain 1728 ſuch little Cubes. *Where note*, that the Beer or Ale Gallon contains 282 cubical Inches, and the Wine Gallon 231; ſo that as often as either of theſe Cubes are contained in any other Number of cubical Inches, ſo many Beer or Wine Gallons are contained therein.

*Where note alſo*, That it was Enacted in the Act of Laying a Duty upon Malt, that every round Buſhel with a plain and even Bottom, being 18,5 Inches Diameter (from Top to Bottom) and 8 Inches Deep, ſhall be eſteemed a Legal *Wincheſter* Buſhel, according to the Standard in her Majeſty's *Exchequer*. Now, if we would find a Diviſor for a Malt Buſhel, it muſt be done by finding the Solid Content of that Cylinder, whoſe Diameter is 18,5 Inches, and Depth 8; and that will be 2150,419, *&c.* And this is the Diviſor for all right-lined Figures: And if the Proportion between this and the circular Diviſor is, as 1,570796 to 2, (as it is) the Circular Diviſor will be 2737,999. But Mr. *Everard* ſays, that 2150, and 2737,47 is ſufficient in Practice.

185
185
9 5
3530
36200
29600

## To Gauge a *Square*.

### *DEFINITION.*

A Square has all its Sides equal, and its Angles right.

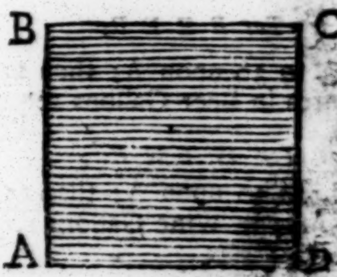
### *RULE.*

Square the Side, and divide the Square by 282 for Beer Gallons, 231 for Wine, and 2150 for Malt.

### *EXAMPLE.*

Let the Side BC of the Square ABCD be 30 Inches; I demand the Area in Beer, Wine, and Malt.

### *FIGURE.*



For,

### *For* BEER.

Side=30
—
30
282) 900,000 (3,191 Area in Beer Gallons at one Inch Deep.
054828
2543
01

### *For* WINE.

231) 900,000 (3,896 Area at one Inch Deep in Wine Gallons.
207214
2242
10

## *For MALT BUSHELS.*

2150) 900,0000 (,4186 Area in Malt.
0400000
18500
131
0

## *By the Sliding Rule.*

### *For* BEER.

Set 282 upon B, to 30 upon A; then againſt 30 on B, is 3,19 on A, the Area in Beer Gallons required.

### *For* WINE.

Set 231 upon B, to 30 upon A; then againſt 30 on B, is 3,89 on A, the Area in Wine Gallons required.

### *For* M A L T.

Set 2150 upon B, to 30 on A; then againſt 30 on B, is ,4186 on A, the Area in Malt. Here you are to know, that this may be called the Rule of Three by the Lines. For as 282, is to 30, ſo is 30 to 3,191, *i. e.* in that Rule you muſt ſet the firſt Number in the Proportion on B, to the ſecond on A; then againſt the third on B, you'll find the fourth required on A, *&c.*

## To Gauge a *Parallelogram.*

### *DEFINITION.*

A Parallelogram is a four-ſided Figure, whoſe two oppoſite Sides (ab, cd or ac, bd) are parallel Lines.

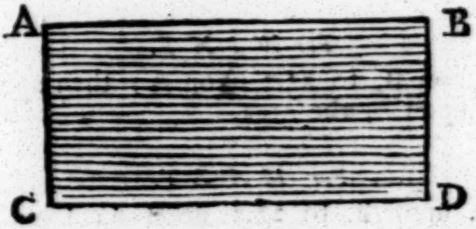
### *RULE.*

Multiply the Length by the Breadth; then divide the Product by 282 for Beer Gallons, 231 for Wine, and 2150 for Malt Buſhels.

### *EXAMPLE.*

Let ABCD be a Parallelogram, whoſe Length AB or CD is 40 Inches, and its Breadth AC or BD 22,3; what's the Area in Beer, Wine, and Malt?

### *FIGURE.*



### *For* BEER.

Breadth 22,3
Length 40

282) 892,0000 (3,1631, the Area required.
0468848
17835
000

### *For* WINE.

231) 892,0000 (3,8614 Area.
199249(6
1430(6
1(1

### *For* MALT.

2150) 892,0000 (,4148 Area.
32000(0
1050(0
12(8
(1

## *By the Sliding Rule.*

### *For* BEER.

Set 282 upon B, to 22,3 upon A; then againſt 40 upon B, is 3,163 upon A; which is the Area required.

### *For* WINE.

Set 231 upon B, to 22,3 upon A; then againſt 40 on B, is 3,861 upon A, the Area.

### *For* MALT.

Set 2150 upon B, to 22,3 upon A; then againſt 40 on B, you'll have ,4148 on A, the Area in Malt.

To

## To Gauge a *Rhombus.*

### *DEFINITION.*

A Rhombus has equal Sides, but unequal Angles.

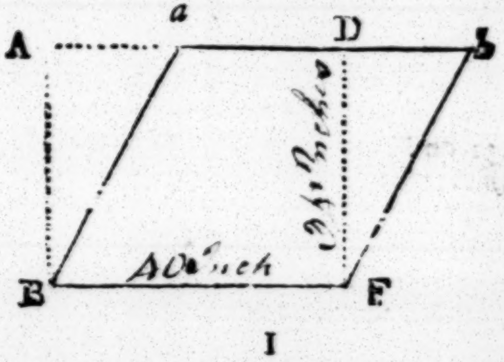
### *RULE.*

It is demonſtrated by the 35th Propoſition of the firſt Book of *Euclid*, that Parallelograms having the ſame Baſe, and being between the ſame Parallels are equal, *i e.* that the Parallelogram ABDF is equal to the Rhombus abBF; therefore, if you multiply BF by BA or DF, the Product will be the Area in Inches; which divide by 282 for Beer Gallons, 231 for Wine, and 2150 for Malt Buſhels.

### *EXAMPLE.*

Let abBF be a Rhombus, whoſe Side BF is 40 Inches, (=AD=ab=bF,) and let the neareſt Diſtance of any two oppoſite Sides *e. g. ab* and BF, *i.e.* DF=AB be 34 Inches; I demand the Area in Beer, Wine, and Malt.

### *FIGURE.*



### *For* BEER.

BF=40
DF=34  *Gall.*
282) 1360,000 (4,822 Area.
232466
9679
01

### *For* WINE.

231) 1360,000 (5,887 Area.
205223
2070
11

### *For* MALT.

2150) 1360,0000 (,6325 Area.
0700000
05505
122
1

## *By the Sliding Rule.*

### *For* BEER.

Set 282 upon B, to 40 on A; then againſt 34 on B, is 4,822 on A; the Area as above.

### *For* WINE.

Set 231 on B, to 40 on A; then againſt 34 on B, is 5,887 on A; the Area as above.

### *For* MALT.

Set 2150 upon B, to 40 on A; then againſt 34 on B, is ,6325 on A; the Area as above.

To

## To Gauge a *Triangle.*

### *DEFINITION.*

¶ Triangles have various Species, firſt [illegible]lation to their Sides, one is called Equilateral, [illegible]

Another Equicrural having two [illegible] equal: And a third Scalenum, becauſe all its Sides are unequal; and in general, a Triangle is a Figure bounded with three right Lines.

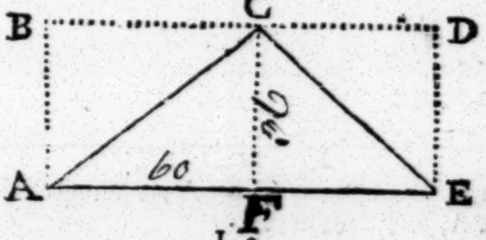
### *RULE.*

*Euclid* in the 41ſt of his firſt Book proves, that a Parallelogram will be double to a Triangle, if they be between the ſame Parallels, and have equal Baſes; as the Triangle ACE is but Half the Parallelogram BDEA, for the Triangle ACF is but Half the Parallelogram BCAF, and the Triangle FCE is Half the Parallelogram FCDE; therefore ACE=Half BDEA. Q. E. D.

Therefore multiply the Baſe AE by half the Perpendicular FC, and the Product is the Area in Inches; which divide by 282 for Beer Gallons, 231 for Wine, and 2150 for Malt.

### *EXAMPLE.*

Let ACE be a Triangle, whoſe Baſe AE is 60 Inches, and Perpendicular FC=36; what's the Area in Beer, Wine, and Malt?



### *For* BEER.

AE the Baſe=60
half the Perp. FC=18

282) 1080,000 (3,829 Area. ✓
234462
0872
22

### *For* WINE.

231) 1080,000 (4,675 Area. ✓
156435
1727
10

### *For* MALT.

2150) 1080,0000 (,5023 Area. ✓
0050000
5005
075
0

## *By the Sliding Rule.*

### *For* BEER.

Set 282 upon B, to 60 on A; then againſt 18 on B, is 3,829 on A, the Area as above.

### *For* WINE.

Set 231 on B, to 60 on A; then againſt 18 on B, is 4,675 on A, the Area as above.

### *For* MALT.

Set 2150 upon B, to 60 on A; then againſt 18 on B, is ,5023 on A, the Area as above.

## To Gauge a *Trapezium.*

### *DEFINITION.*

A *Trapezium* is a Figure conſiſting of four unequal Sides, as ABCD.

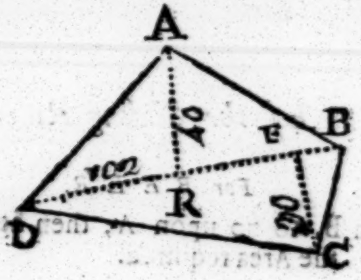
### *RULE.*

Having divided the *Trapezium* into Triangles, as in the Figure following, add the two Perpendiculars, *viz.* AR and FC together, and multiply that Sum by Half the Baſe BD, and the Rectangle or Product is the Area in Inches; which divide by 282 for Beer Gallons; 231 for Wine, and 2150 for Malt Buſhels.

### *EXAMPLE.*

Let ABCD be a *Trapezium,* let the common Baſe DB be 102 Inches, the Perpendicular AR 40, and FC = 30; I demand the Area in Beer, Wine, and Malt.

*FIGURE.*



### *For* BEER.

AR=40
FC=30

Sum=70
½DB=51

282) 3570,000 (12,659 Area.
0756802
18676
121

### *For* WINE.

231) 3570,000 (15,454 Area.
1265656
10202
111

### *For* MALT.

2150,) 3570,000 (1,660 Area.
1420000
13000
010
1

## *By the Sliding Rule.*

### *For* BEER.

Set 282 on B, to 70 upon A; then against 51 on B. is 12,659 on A; the Area required.

### *For* WINE.

Set 231 on B, to 70 upon A; then against 51 on B, is 15,454 on A; the Area required.

*For* MALT.

Set 2150 upon B, to 70 on A; then againſt 51 on B, is 1,660 upon A; the Area as above.

## To Gauge a *Hexagon*.

DEFINITION.

A *Hexagon* is a *Polygon* of ſix equal Sides, and as many Angles; as ABCDEF.

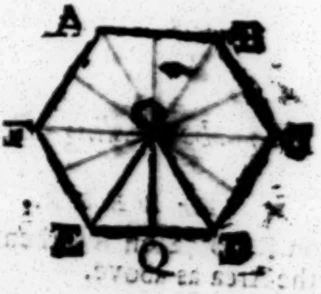
RULE.

All *Polygons* may be meaſured by one general Rule; *viz.* Multiply the Sum of all their Sides by Half the Diſtance of any Side from the Center, and divide the Product by 282 for Beer Gallons, 231 for Wine, and 2150 for Malt.

EXAMPLE.

Let ABCDEF be a *Hexagon*; any of whoſe Sides is 12 Inches, and the neareſt Diſtance from the Center to the Middle of any Side, as OQ is 11,4 Inches; what's the Area in Beer, Wine, and Malt?

*FIGURE.*



### *For* B E E R.

12
6

Sum of the Sides = 72
Half the Perpendicular 5,7

504
360

282) 410,400 (1,455 Area.
128600
1559
10

### *For* W I N E.

231) 410,400 (1,776 Area.
179734
1754
11

### *For* M A L T.

2150,) 410,4000 (,1908 Area.
954000
01900
198
1

### *By the Sliding Rule.*

### *For* B E E R.

Set 282 upon B, to 72 on A; then againſt 5,7 on B, is 1,455 on A; the Area as above.

1·455

For

### *For* WINE.

Set 231 upon B, to 72 on A; then againſt 5,7 on B is 1,776 on A; the Area as above.

### *For* MALT.

Set 2150 upon B, to 72 on A; then againſt 5,7 on B, is ,1908 on A; the Area as above.

## To Gauge a *Circle*.

### DEFINITION.

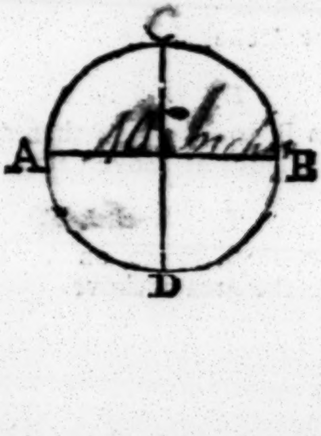
A Circle is a plain Figure, terminated by the incompaſſing of one Line, which is called the Circumference, and is every where equally remote from the middle Point or Center.

### RULE.

Divide the Square of the Diameter of any Circle by 359 for Beer Gallons, 294 for Wine, and 2737,47 for Malt.

### EXAMPLE.

Let ABCD be a Circle, whoſe Diameter AB is 40 Inches; what's the Area in Beer, Wine, and Malt?



### *For* BEER.

The Diameter=40 Inches.
40

359) 1600,000 (4,456 Area.
164456
2049
22

### *For* WINE.

294) 1600,0000 (5,4421 Area.
130442(6
1265(2
00(2

### *For* MALT.

2737,47) 1600,00000 (,584 Area.
23126542
122675
1317

## *By the Sliding Rule.*

### *For* BEER.

Set 359 upon B, to 40 on A; then againſt 40 on B, is 4,456 on A; the Area as above.

### *For* WINE.

Set 294 upon B, to 40 on A; then againſt 40 on B, is 5,4421 on A, the Area as above.

### *For* MALT.

Set 2737,47 upon B, to 40 on A; then againſt 40 on B, is ,584 on A, the Area as above.

To

## To Gauge a *Sector* of a Circle.

### *DEFINITION.*

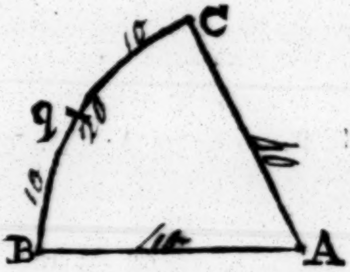
A Sector of a Circle is contained upon the Ends of two right Lines or Semi-diameters, making an Angle in the Center, having any Part of the Circumference for its Baſe.

### *RULE.*

Multiply the *Radius* or Semi-diameter AB or AC by Half the Arch of the Section BC. *i. e.* C*q*, and divide the Product by 282 for Beer Gallons, 231 for Wine, and 2150 for Malt.

### *EXAMPLE.*

Let A B *q* C be a Sector of a Circle, and let the *Radius* A B = AC be 40 Inches, and BC = 20, then is B*q* = C*q* = 10; I demand the Area in Beer, Wine, and Malt.



*For* BEER.

Radius AB or AC = 40
Half the Arch Line BC = 10

282) 400,000 (1,418 Area.
- 118284
0532
21

For

### *For* WINE.

231) 400,000 (1,731 Area.
16937(9
73(3
(1

### *For* MALT.

2150) 400,0000 (,1860 Area.
1850000
13000
10
10

## *By the Sliding Rule.*

### *For* BEER.

Set 282 upon B, to 40 on A; then againſt 10 on B, is 1,418 on A; the Area as above.

### *For* WINE:

Set 231 upon B, to 40 on A; then againſt 10 on B, is 1,731 on A; the Area as above.

### *For* MALT.

Set 2150 upon B, to 40 on A; then againſt 10 on B, is ,1860 on A; the Area as above.

## To Gauge an *Oval.*

### DEFINITION.

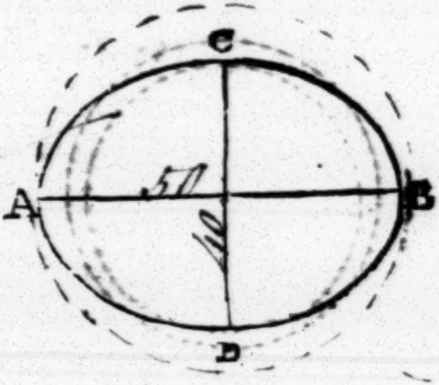
An Oval or Ellipſis is a circular Figure, as ABCD, having two unequal Diameters, as AB, which is called the Tranſverſe, and CD the conjugate Diameter, and proved by *Archimedes* to be a mean Proportional between the two Circles that may be deſcribed ſeverally on theſe Diameters.

### *RULE.*

Multiply the tranſverſe Diameter AB, by the conjugate Diameter CD; then divide the Product by 359 for Beer Gallons, 294 for Wine, and 2737,47 for Malt Buſhels.

### *EXAMPLE.*

Let the Figure ABCD be an Oval or Ellipſis, whoſe conjugate Diameter CD let be 40 Inches, and tranſverſe Diameter AB 50; what's the Area thereof in Beer, Wine, and Malt.



*For BEER.*

AB=50
CD=40

359) 2000,000 (5,571 Area.
205571
2531
00

*For* W I N E.

294) 2000,000 (6,802 Area.
236802
0081
02

*For* M A L T.

2737,47 (2000,0000 (,73 Area.

## *By the Sliding Rule.*

*For* B E E R.

Set 359 upon B, to 50 on A; then againſt 40 on B, is 5,571, the Area as above. on A.

*For* W I N E.

Set 294 upon B, to 50 on A; then againſt 40 on B, is 6,802 on A, the Area as above.

*For* M A L T.

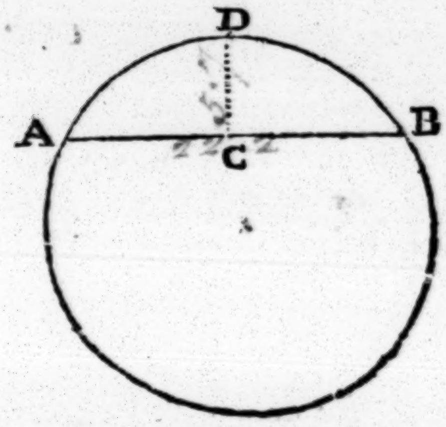
Set 2737,47 upon B, to 50 on A; then againſt 40 on B, is ,73 on A, the Area as above.

## To Gauge the *Segment* of a *Circle.*

*DEFINITION.*

A Segment of a Circle is a Figure contained between a right Line AB (called a Chord) and any Part of a Circumference called an Arch, as ADB.

*RULE.*



B

### *RULE.*

Multiply the Chord AB by two Thirds of the verſed Sine DC, and the Product will be very near the Area in Square Inches; which divide by 282 for Beer Gallons, 231 for Wine Gallons, and 2150 for Malt Buſhels.

### *EXAMPLE.*

ABD in the preceding Figure, is a Segment of a Circle, whoſe Chord Line AB is 22,2, and verſed Sine DC is 5,7 Inches; what's the Area thereof in Beer, Wine, and Malt?

*For BEER.*

DC=5,7 ✓
Two Thirds is 3,8 ✓
AB 22,2 ✓
1776 ✓
666
282) 84,360 (,299 Area. Beer
2798(2
25(4
(0

3.7
12
3 [illegible].41
.3.8

*For* W I N E.

```
231,)  84,360  (,365 Area.
       15005
        124
         0
```

*For* M A L T.

```
2150)  84,3600  (,0392 Area.
       19860(0
        051(0
          (8
```

## *By the Sliding Rule.*

Firſt to find $\frac{2}{3}$ of DC=5,7: Set 3 the Denominator on B, to 2 the Numerator on A; then againſt the verſed Sine 5,7 on B, you'll find 3,8 on A, which is $\frac{2}{3}$ of 5,7 as above.

*For* B E E R.

Set 282 upon B, to 3,8 upon A; then againſt 22,2 upon B, you'll find, ,299, the ~~Area~~ as above. on A

*For* W I N E.

Set 231 upon B, to 3,8 upon A; then againſt 22,2 on B, you'll find ,365 on A, the Area as above.

*For* M A L T.

Set 2150 upon B, to 3,8 on A; then againſt 22,2 on B, you'll find ,0392 on A, the Area as above.

## *To find the Area of an irregular Superficial Figure.*

Divide it into Triangles, Trapeziums, *&c.* as the Nature of the Figure will bear; then find the Contents of thoſe ſeveral Parts in Inches, and add them together; which Sum divide by 282 for Beer Gallons, 231 for Wine, and 2150 for Malt Buſhels.

## *EXAMPLE.*

Let ABCDEGF be an irregular Figure, whoſe Area in Beer and Wine Gallons is required; this Figure, when divided, as you ſee, is two Trapeziums, and one Triangle, whoſe Dimenſions you may alſo ſee in the ſame Figure.

Find their Areas, as you did thoſe of the Trapezium and Triangle, (at *pag.* 90, and 88,) which will be

| | *Inches.* |
|---|---|
| Area of the Trapezium FABC is | 560 |
| Area of the Trapezium FCDE is | 480 |
| Area of the Triangle FEG is | 170 |
| Their Sum is | 1210 |

*For* B E E R.

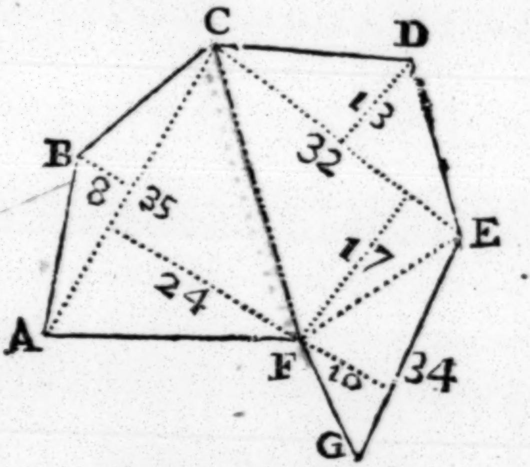
282) 1210,00(0 (4,29 Area.
0826(2
25(2
(0

*For* W I N E.

231) 1210,000 (5,238 Area.
05587(2
088(2
1(0

*For* ***MALT.***

```
          L
2150,)  1210,0000  (,5627 Area in Malt Buſhels.
        135000(0
         0600(5
          17(9
           (1
```



L

### *EXAMPLE* II.

Let ABCDEFGHIKLMNP be an irregular Figure, which being divided into convenient Parts, will appear as you ſee, and the Content of each Part be as follows.

| | |
|---|---|
| The Area of the Triangle, EPM is | 63 |
| The Area of the Trapezium, EPAD is | 260 |
| The Area of the Trapezium, ABCD is | 154 |
| The Area of the Trapezium, EFGK is | 442 |
| The Area of the Segment, HIKQ is | 40 |
| The Area of the Trapezium, KLME is | 169 |
| Their Sum is | 1128 |
| The Area of the Segment, MNPO is | 24 18 |
| The Remainder is | ~~1104~~ 1110 which is |

Area of Segment H. I. K. add

The Area of the whole Figure in Square Inches, which divide as below.

*For* BEER.

282) 1104,000 (3,9148 Area.

*For* WINE.

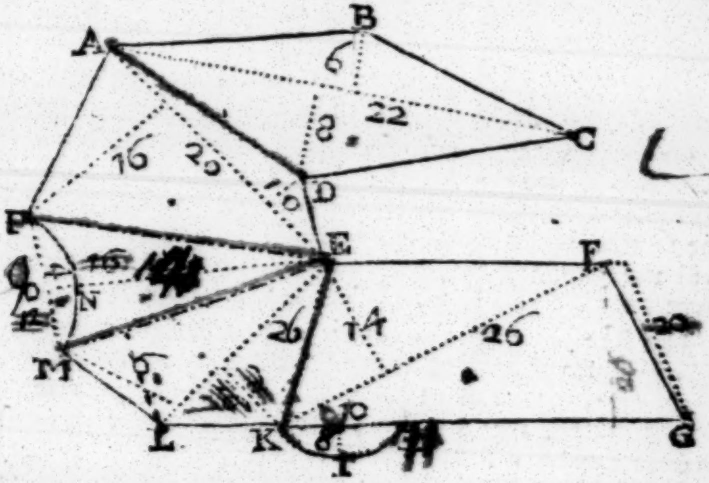
231) 1104,0000 (4,7792 Area.

*For* MALT.

2150) 1104,0000 (,5134 Area.

In the Triangle EMP, MP is 9 Inches, and EO is 14 Inches.

In the *Trapezium* KLME, let the Perpendicular from M be 6 Inches, and the Perpendicular from K 7 Inches, and let the Base LE be 26 Inches.



NOTES

## NOTES *upon the foregoing Examples.*

*1ſt*, That if the Sides of any Tun are parallel, and that it has any of the preceding Figures for its Baſe; the Content of all ſuch Tuns may be found by multiplying the Area of their Baſes (found as above directed) by the Depth, and this will always hold; let the Area of the Baſe be found in what Denomination ſoever, *i. e.* whether in Feet, Inches, Ale, Wine, or Malt, *&c.*

*2dly*, That when you are to divide by 282 for Beer Gallons, and 231 for Wine, you may multiply the intended Dividend by ,0035461, and the Product will be the Content in Beer Gallons, and by ,004329, and the Product will be the Content in Wine Gallons.

*3dly*, That when you are to divide by 359 for Beer Gallons, and 294 for Wine, (as in Circles, *&c.*) you may multiply the intended Dividend by ,0027855, and the Product will be the Content in Beer Gallons, and by ,0034, and the Product will be the Content in Wine Gallons.

*4thly*, That when you are to divide by 2737,47, the Diviſor for a Circle in Malt Buſhels.

Or by 2150 the Diviſor for Squares, *&c.* you may multiply the Square of the Diameter of a Circle, or any other proper Dividend to the Diviſor, by ,0003653. And the Square of the Side of a Square, *&c.* by ,0004651, ſo will the Products be the ſame as the Quotients would have been, if Diviſion had been made.

The Reaſon why the Multiplicators before-mentioned will perform the ſame as the Diviſors, may be deduced from *Prop.* 33. *Lib.* 1 *Matheſ. Enucleata*, where 'tis demonſtrated, that ſimilar plane Figures are to one another in duplicate Proportion of their homologous Sides; and alſo from *Euclid*, *Prop.* 2. *Lib.* 12. where 'tis demonſtrated, mon[illegible]ted, that Circles are in the ſame Proportion as the Squar[illegible]f their Diameters.

Now [illegible]ce Squares are to one another, as the Squares of their Sides; let us ſuppoſe the Side of a Square to be 1 Inch, then by the preceding Rule for that purpoſe, the Area of ſuch a Square in Beer Gallons will be ,0035461, and in Wine ,004329; therefore, as the Square of 1, *i. e.* but 1, is to its Area, *viz.* ,0035461 in Beer, or ,004329 in Wine, ſo is the Square of the Side of any other Square, to the Area thereof in Beer or Wine. Now according to the Rule of Proportion, we ought to multiply the ſecond and third Numbers together, and divide the Product by the firſt, but becauſe in this Caſe the firſt Number is but 1, the Product muſt be the Anſwer; for 1 will neither multiply nor divide.

The ſame is to be ſaid of Circles; ſince (as above) they are to one another in the ſame Proportion as the Squares of their Diameters; if we ſuppoſe the Diameter of a Circle to be 1, then by the preceding Rule for that purpoſe, the Area in Beer will be ,0027855, and in Wine ,0034. Therefore, as the Square of the Diameter, *viz.* 1, *i. e.* but 1, is to its Area ,0027855 in Beer, or to ,0034 in Wine, ſo is the Square of any other Diameter to the Area of its Circle; and here alſo, becauſe 1 is the firſt Number of the Proportion, and can't divide, the Product muſt be the Anſwer; and the ſame Reaſon ſerves for the Malt Multiplicators.

## To Gauge a *Cube.*

### *DEFINITION.*

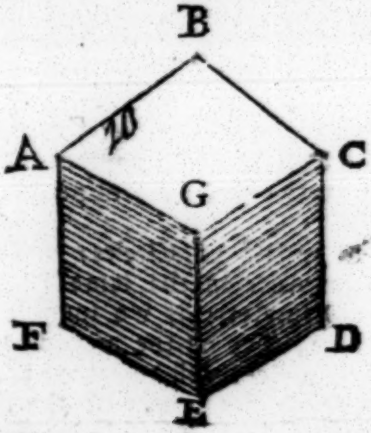
A Cube (or Dye) is a Solid Figure contained under 6 equal Squares.

### *RULE.*

Cube the Side, (*i e* multiply the Side of the Cube by it ſelf, and that Product again by the Side of the Cube) then divide that cubed Number by 282 for Beer Gallons, 231 for Wine, and 2150 for Malt.

*EXAMPLE.*

Let ABCDEFG repreſent a Cube, whoſe Side let be 20 Inches; I demand the Content in Beer and Wine Gallons. and Malt Bushels.



Side=20
20

400 its Square.

20
282) 8000,00 (28,36 Beer.
236448
1094
12

For *W I N E.*

231) 8000,00 (34,63 Content in Wine.
107647
1474
00

### *For* MALT.

2150,) 8000,00 (3,720 Content in Malt.

## By the *Sliding Rule.*

To perform this Queſtion by the Inſtrument at one Operation, there ought to be inſerted in the Line D. three Gauge Points, *i. e.* one at 16,79, the Square Root of 282 for Beer Gallons; one at 15,19, the Square Root of 231 for Wine, and one at 46,36, the Square Root of 2150 for Malt Buſhels; and their Uſe is as below.

### *For* BEER.

Set 16,79 upon D, to 20 on C; then againſt 20 on D, is 28,36, the Content in Beer as above.

### *For* WINE.

Set 15,19 upon D, to 20 on C; then againſt 20 on D, is 34,63 on C, the Content in Wine as above.

### *For* MALT.

Set 46,36 upon D, to 20 on C; then againſt 20 on D, is 3,720 on C, the Content required.

## To gauge a *Parallelopipedon.*

### DEFINITION.

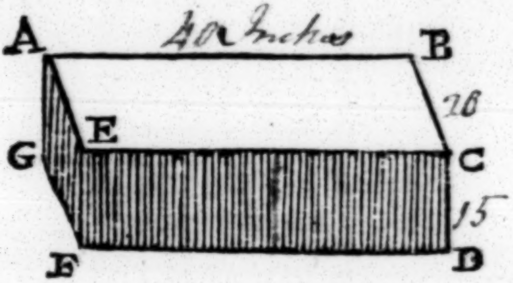
A Parallelopipedon is a Solid Figure contained under ſix Parallelograms, the oppoſite of which are Parallel.

### RULE.

Multiply the Length by the Breadth, and the Product by the Depth; then divide by 282 for Beer Gallons, 231 for Wine, and 2150 for Malt Buſhels.

### EXAMPLE.

Let ABCDEFG repreſent a Parallelopipedon, whoſe Length AB is 40 Inches, the Breadth BC = 20 Inches, and Depth CD = 15; what's the Content in Beer, Wine, and Malt?



### For BEER.

Length 40
Breadth 20

800
Depth 15

282) 12000,00 (42,55 Beer Gallons.
072600
1559
10

### For WINE.

231) 12000,00 (51,94 Content in Wine.
045916
2118
11

### For MALT.

2150,) 12000,000 (5,581 in Malt Buſhels.

By

## *By the Sliding Rule.*

Firſt, by *Prob.* 1ſt of the Uſe of the Square Root by the Lines C and D, (in the preceding Part) find a mean Proportional between the Length AB = 40, and the Breadth BC = 20; which according to the Rule there given, you'll find to be 28,28 Inches. Then

### *For BEER.*

Set the Gauge Point 16,79 on D, to 15 the Depth on C; then againſt 28,28 (the Mean) on D, you'll find 42,55 on C, the Content as above.

### *For WINE.*

Set the Gauge Point 15,19 on D, to 15 the Depth on C, then againſt 28,28 (the Mean) on D, you'll find 51,94, the Content on C.

### *For MALT.*

Set the Gauge Point 46,36 on D, to 15 on C; then againſt 28,28 on D, is 5,581 on C; the Content required.

## To gauge a *Priſm.*

### *DEFINITION.*

A Priſm is a Solid Figure, the two oppoſite Ends of which are equal alike and parallel; and all other Sides are Parallelograms.

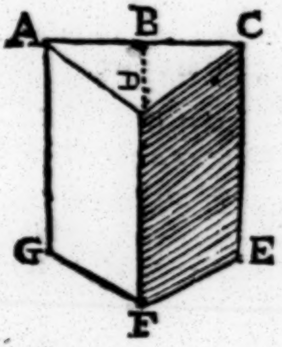
### *RULE.*

Multiply the Baſe of the Triangle at the End by Half the Perpendicular, then multiply that Product by the

Depth, and divide the laſt Product by 282 for Beer, 231 for Wine, and 2150 for Malt.

### *EXAMPLE.*

Let ABCDEFG repreſent a Triangular Priſm, (for there are diverſe Sorts of them, but all take their Denomination from their Baſes or Ends;) let AC be 40 Inches, BD = 22 Inches, and the Depth AG = 60, what's the Content in Beer and Wine Gallons, and Malt Buſhels?



Thomas
Loves
Smith

*For* BEER.

AC = 40

Half BD = 11

440

Depth 60

282) 26400,00 (93,61 Content in Beer Gallons.

102488

1749

01

*For* WINE.

231) 26400,00 (114,28 Content in Wine
0339682 Gallons.
09693
011

*For* MALT.

2150,) 26400,00 (12,27 Content in Malt.

## *By the Sliding Rule.*

Firſt find a mean Proportional between Half BD = 11, and AC = 40 ; which by the Sliding Rule is 20,97. Then

*For* BEER.

Set the Gauge Point 16,79 on D, to 60 on C; then againſt 20,97 on D, is 93,61 on C, the Content as above.

*For* WINE.

Set the Gauge Point 15,19 on D, to 60 on C; then againſt 20,97 on D, is 114,28 on C, as above.

*For* MALT.

Set the Gauge Point 46,36 on D, to 60 on C; then againſt 20,97 on D, is 12,27 on C, the Content as above.

## To gauge a *Cylinder*.

*DEFINITION.*

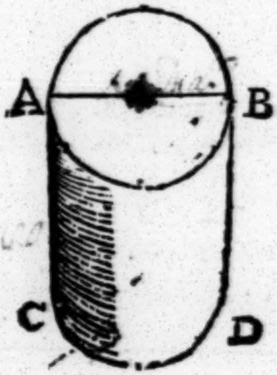
A Cylinder is a Solid Figure, riſing from a circular Baſe (or Circle) in right Lines, and ending in an equal Circle, or 'tis like the Rowling-Stone of a Garden.

### *RULE.*

Square the Diameter, and multiply that Square by the Depth; then divide by 359 for Beer Gallons, 294 for Wine, and 2737,47 for Malt Buſhels.

### *EXAMPLE.*

Let ABCD repreſent a Cylinder; and let AB (the Diameter) be 40 Inches, and AC the Depth = 60; what's the Content thereof in Beer, Wine, and Malt?



The Diameter 40 = AB
40

its Square = 1600
the Depth AC = 60

Product 96000 which divide by the proper Diviſors.

*For* BEER.

359) 96000,000 (~~3~~67,409 Content.
24267409
264346
1031

267.409 DDH

*For*

### *For* W I N E.

294) 96000,000 (326,530 Content.
07826080
195928
1002

### *For* M A L T.

2737,47) 96000,0000 (35,068 Content.

## *By the Sliding Rule.*

Upon the Line D you have three Gauge Points; the firſt mark'd with *W. g.* for Wine Gallons, the ſecond with *A. g.* for Ale Gallons, and the third with *M. g.* for the Malt Gauge Point; their Uſe is as below.

### *For* B E E R.

Set *A g.* = 18,94 on D, to the Depth 60 upon C, then againſt the Diameter = 40 on D, you'll find 267,409 as above.

Exd. DDH

### *For* W I N E.

Set *Wg.* = 17,14 on D, to the Depth = 60 on C; then againſt the Diameter 40 on D, you'll find 326,530, as above.

Exd. DDH

### *For* M A L T.

Set *M g.* = 52,32 on D, to 60 the Depth on C; then againſt 40 on D, is 35,068 on C, for the Content as above.

Exd. DDH

## To Gauge a *Cone.*

### D E F I N I T I O N.

A Cone is a ſolid Figure, riſing from a circular Baſe

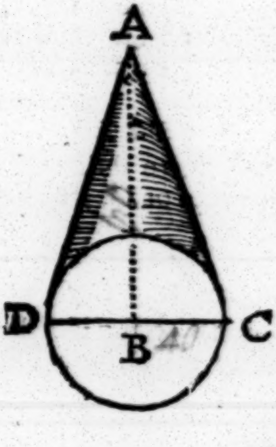
(CD) in right Lines, (CA DA) and ending in a Point, (A) which is called the Vertex or Top.

### *RULE.*

Square the Diameter, and multiply that Square by the Heighth; then divide by 1077 for Beer Gallons, 882 for Wine, and 8212,41 for Malt Buſhels.

### *EXAMPLE.*

Let ABCD be a Cone, whoſe Heighth AB is 60 Inches, and Diameter DC 40; I demand the Content in Beer, Wine, and Malt.



Diameter DC = 40
40

its Square 1600
the Heighth 60

the Square of the Diameter in the Heighth 96000,
which divide by 1077, by 882, and by 8212,41 for Beer, Wine, and Malt, as below.

*For* BEER.

1077) 96000,000 (89,136 the Content. Exd
09847398
014992
0365
00

*For* WINE.

882) 96000,000 (108,843 Content. Exd

*For* MALT.

8212,41) 96000,00 (11,68 Content. Exd.

## *By the Sliding Rule.*

*For* BEER.

Set the Gauge Point *Ag* = 18,94 on D, to one third of the Heighth, *i. e.* 20 on C: then againſt the Diameter, *viz.* 40 on D, you'll have 89,136 as above. Exd.

*For* WINE.

Set the Gauge Point *W. g.* on D, (*viz.* 17,14) to 20 (=$\frac{1}{3}$ of 60) on C; then againſt the Diameter, *viz.* 40 on D, you'll find 108,843 upon C, as above. Exd.

*For* MALT.

Set the Gauge Point *Mg.* (52,32) on D to 20 (=$\frac{1}{3}$ of 60) on C; then againſt the Diameter, *viz.* 40 on D, is 11,68 on C, as above. Exd. DDH

## To gauge a Square *Pyramid.*

### DEFINITION.

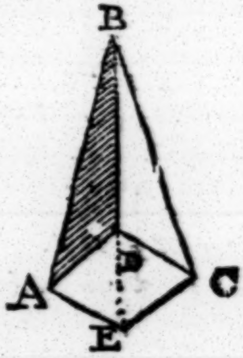
A Square Pyramid is a ſolid Figure, whoſe Sides are plain Triangles, and ariſing from a Square Baſe, meet in one Point at B.

### RULE.

Square the Side, and multiply that Square by the Heighth; then divide by 846 (= 3 Times 282) for Beer Gallons, 693 (= 3 Times 231) for Wine, and 6451,26 (= 3 Times 2150,42) for Malt Buſhels.

### EXAMPLE.

Let ABCDE, repreſent a ſquare Pyramid, whoſe Side let be 40 Inches, and Heighth 60; what's the Content in Beer, Wine, and Malt?



The Side AE = CE = DC = 40 Inches.
40

Its Square is 1600
Which multiply by 60 B D

and the Product 96000 is a Dividend, which if you divide by 846,693, and 6451,26 (as by the Rule) the Quotients will be the Content in Beer, Wine, and Malt.

### *For* BEER.

846) 96000,000 (113,475 Content.
11442680
290335
4641

### *For* WINE.

693) 96000,000 (138,528 Content.
26716546
596969
3150

### *For* MALT.

6451,26) 96000,000 (14,88 Content.

## *By the Sliding Rule.*

### *For* BEER.

Set 16,79 (the Gauge Point for a Square in Beer Gallons) on D, to 20 ($=\frac{1}{3}$ of 60) upon C; then againſt 40 (the Side given) on D, is 113,475, for the Content in Beer Gallons and Parts, as above on C.

### *For* WINE.

Set 15,19 (the Gauge Point for a Square in Wine Gallons) on D, to 20 ($=\frac{1}{3}$ of 60) upon C; then againſt the given Side, *viz.* 40 on D, you'll find 138,528, the Content in Wine Gallons and Parts, as above on C.

### *For* MALT.

Set 46,36 (the Malt Gauge Point for a Square) on D, to 20 ($=\frac{1}{3}$ of 60) on C; then againſt the given Side, *viz.* 40 on D, you'll find 14,88 on C, the Content in Malt Buſhels and Parts, as above.

To

### DEFINITION.

## To gauge a *Parabolick Conoid.*

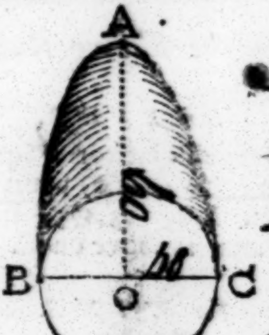
A Parabolick Conoid is made up of an infinite Number of Circles, whoſe Area's are in Arithmetick Progreſſion, and formed by the Rotation of a Semiparabola, *&c.*

### RULE.

Multiply the Square of the Diameter by the Heighth, and divide the Product by 718 for Beer Gallons, ,588 for Wine, and 5474,94 for Malt Buſhels.

### EXAMPLE.

Let ABC be a Parabolick Conoid, whoſe Diameter BC is 40 Inches, and Heighth AO 60; what's the Content thereof in Beer and Wine Gallons, and Malt Buſhels?



B C

Gr

this is

give

### DEFINITION.

| | |
|---|---|
| The Diameter BC is | 40 Inches. |
| | 40 |
| its Square is | 1600 |
| the Heighth AO is | 60 |
| the Rect. of the Heighth in the Sq. of the Diam | 96000 |

which divide according to the Rule, as ~~below~~ above

For

### *For* BEER.

718) 96000,000 (133,704 Content.
24266408
260342
5035

### *For* WINE.

588) 96000,000 (163,265 Content.
37226420
195818
1331

### *For* MALT.

5474,94) 96000,000 (17,53 Anſwer.

## *By the Sliding Rule.*

### *For* BEER.

Set 18,[illegible] (the Beer Gauge Point for a Circle) on D, to 30 [illegible] Half of the Heighth, *viz.* 60) on C; then againſt 40 (the Diameter) on D, you'll find 133,704, the Content in Gallons and Parts on the Line C.

### *For* WINE.

Set 17,14 (the Gauge Point for a Circle in Wine Gallons) on D, to 30 (*i. e.* Half the Heighth, *viz.* 60) on C; then againſt the Diameter, *viz.* 40 on C, you'll find 163,265, the Content required on C.

### *For* MALT.

Set 52,32 (the Malt Gauge Point for a Circle) on D, to 30 (*i. e.* Half the Heighth, *viz.* 60) on C; then againſt 40 (the Diameter) on D, you'll find 17,53, the Content required on C.

## To gauge a *Hyperbolick Conoid.*

### *DEFINITION.*

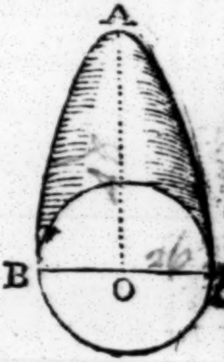
An Hyperbolick Conoid is made by the Rotation of a Semi-hyperbola about its Axe.

### *RULE.*

Multiply the Square of the Diameter by the Heighth, and then divide that Product by 862 for Beer Gallons, by 706 for Wine, and 6569,9 for Malt Buſhels.

### *EXAMPLE.*

Let ABOC repreſent a Hyperbolick Conoid, whoſe Heighth AO let be 48 Inches, and Diameter BC 26; what's the Content in Beer and Wine Gallons, and Malt Buſhels.



The Diameter BC, is - 26 Inches.
26
156
52
its Square is 676
48 = Heighth,
5408
2704
theRect.of theSq.of the Diam.&Heighth is 32448

**For BEER.**

862) 32448,00 (37,64 Content.
658482
5563
32

**For WINE.**

706) 32448,00 (45,96 Content.
420864
6722
40

**For MALT.**

6569,9) 32448,00 (4,938 Content.

## *By the Sliding Rule.*

**For BEER.**

Set 29,36 (the Beer Gauge Point for a Hyperbolick Conoid) on D, to the Heighth, *viz.* 48 Inches on C; then againſt 26 (the Diameter) on D, you'll find 37,64 on C, as above.

**For WINE.**

Set 26,57 (the Wine Gauge Point for a Hyperbolick Conoid) on D, to the Heighth *viz.* 48 Inches on C; then againſt the Diameter, *viz.* 26 on D, you'll find 45,96, the Content required on C.

**For MALT.**

Set 81,05 (the Malt Gauge Point for a Hyperbolick Conoid) on D, to 48 on C; then againſt 26 on D, you'll find 4,938, the Content in Buſhels and Parts, as above.

## To gauge a *Spheroid.*

### *DEFINITION.*

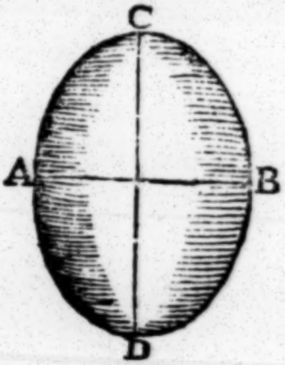
A Spheroid is an oval Solid, longer than it is broad or deep, being made by the Rotation of a Semi-Ellipſis.

### *RULE.*

Multiply the Square of the Conjugate by the tranſverſe Diameter; then divide the Product by 538 for Beer Gallons, 441 for Wine, and 4106,2 for Malt.

### *EXAMPLE.*

Let ABCD be a Spheroid, whoſe conjugate Diameter AB is 42 Inches, and tranſverſe Diameter CD 60; I demand the Content in Beer, Wine, and Malt.



| | |
|---|---|
| | 42 |
| The Conjugate (or ſhorteſt) Diameter, *viz.* AB is | 42 |
| | 84 |
| | 168 |
| Its Square is | 1764 |
| The Tranſverſe (or longeſt) Diameter is DC= | 60 |
| The Dividend is | 105840 |

Which if you divide by the Diviſors as in the Rule, you'll have the Content of the Spheroid in Beer and Wine Gallons, and Malt Buſhels.

*For* BEER.

538) 105840,00 (196,72 the Content in Beer Gallons and Parts.
5202244
36956
314

*For* WINE.

441) 105840,00 (240 the Content in Wine Gallons.
17600
000

*For* MALT.

4106,2) 105840,00 (25,77 the Content in Buſhels and Parts.

## By the *Sliding Rule.*

*For* BEER.

Set 23,19 ( the Beer Gauge Point for a Spheroid ) on D, to the tranſverſe Diameter, *viz.* 60 on C; then againſt the conjugate Diameter, *viz.* 42 on D, you'll find 196,72 on C, the Content required as above.

*For* WINE.

Set 21 (the Wine Gauge Point for a Spheroid) on D, to the tranſverſe Diameter, *viz.* 60 on C; then againſt 42 on D, you'll find 240 on C, for the Content as above.

*For* MALT.

Set 64,07 on D, to 60 on C; then againſt 42 on D, is 25,77, the Content required on C.

## To gauge a *Globe*.

### *DEFINITION.*

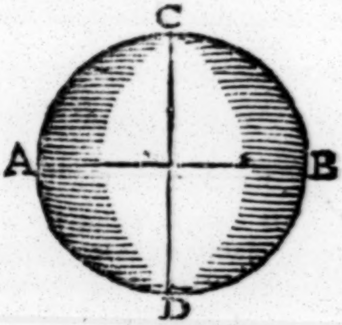
A Sphere or Globe is a round Solid, whoſe Length, Breadth, and Depth is alike or equal.

### *RULE.*

Cube the Globe's Diameter; then divide that Cube by 538 for Beer Gallons, 441 for Wine, and 4106,2 for Malt Buſhels.

### *EXAMPLE.*

Let ABCD repreſent a Globe, whoſe Diameter AB or CD is 36 Inches; I demand the Content in Beer and Wine Gallons, and Malt Buſhels.



The Diameter AB=CD is 36 Inches.
36
216
108
Its Square is 1296
36
7776
3888
Its Cube (for a Dividend) is 46656

*For* BEER.

538) 46656, (86,72, the Content in Beer Gallons and Parts.
36184
386

*For* WINE.

441) 46656.00 (105,79, the Content in Wine Gallons and Parts.
0255131
25526
342

*For* MALT.

4106,2) 46656,0000 (11,36, the Content in Buſhels and Parts.

## *By the Sliding Rule.*

*For* BEER.

Set 23,19 upon D, to 36 upon C; then againſt 36 on D, you'll find 86,72 on C, the Content required.

*For* WINE.

Set 21 on D, to 36 on C; then againſt 36 on D, you'll find 105,79 on C, the Content required.

*For* MALT.

Set 64,07 on D, to 36 on C; then againſt 36 on D, you'll find 11,36 on C, the Content required; where you may obſerve, that the ſame Diviſor and Gauge Point ſerves both the Globe and Spheroid, they being the ſame Part of their circumſcribing Cylinder.

## To gauge an *elliptick Cone.*

### DEFINITION.

An elliptick Cone is a ſolid Figure, ariſing from an

34.6

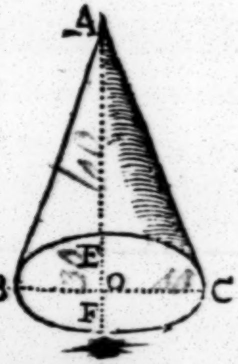
elliptical Baſe in ſtraight Lines, and ending in a Vertex or Point as A.

### *RULE.*

Multiply the Rectangle of the bottom Diameters by the Heighth; then divide by 1077 for Beer Gallons, 882 for Wine, and 8212,41 for Malt Buſhels.

### *EXAMPLE.*

Let BACO repreſent an upright elliptick Cone; the conjugate Diameter of its Baſe is 30 Inches, the Tranſverſe 40, and Heighth AO 60; what's the Content in Beer, Wine, and Malt?



| | |
|---|---|
| The Tranſverſe Diameter BC is | 40 Inches. |
| The Conjugate Diameter EF is | 30 |
| Their Rectangle is | 1200 |

Which multiply'd by the Alt. AO=60, gives 72000 for a Dividend.

### *For* BEER.

1077) 72000,00 (66,85 the Content in Beer Gallons and Parts.
0738845
09165
052
0

*For*

*For* WINE.

882) 72000,00 (81,63, the Content in Wine
144884 Gallons and Parts.
5583
22

*For* MALT.

8212,41) 72000,0000 (8,767 the Content in
Malt Buſhels and Parts.

### *By the Sliding Rule.*

*For* BEER.

Firſt, find a mean Proportional between the Diameters, *viz.* 30 and 40, which (by the Sliding Rule) you'll find to be 34,64: Then ſet 32,82 (the Beer Gauge Point for round Pyramids) on D, to 60 (the Heighth) on C; then againſt 34,64 (the mean Diameter) on D, you'll find 66,85 (as above) on C.

*For* WINE.

Set 29,7 (the Wine Gauge Point for round Pyramids) on D, to 60 on C; then againſt 34,64 (the Mean) on D, you'll find 81,63 on C, as above.

*For* MALT.

Set 90,62 on D, to 60 on C; then againſt 34,64 on D, you'll find 8,767 on C, as above.

## To gauge the *Fruſtum* of a Cone.

*DEFINITION.*

If a Cone, Conoid, or Pyramid, be cut by a plain Parallel to the Baſe; that which remains below, is called the Fruſtum of a Cone, Conoid, &c.

RULE

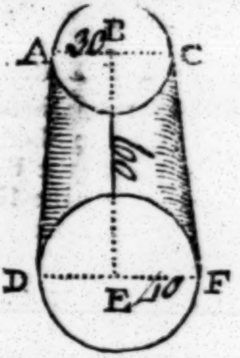
1.43
2.
3.35

### *RULE.* I.

To three Times the Rectangle of the Diameters, add the Square of their Difference, multiply the Sum by the Depth, and divide by 1077 for Beer Gallons, 882 for Wine, and 8212,41 for Malt Buſhels.

### *EXAMPLE.*

Let ABCDEF, repreſent the Fruſtum of a Cone; let the Top Diameter, *viz.* AC be 30 Inches, the Bottom Diameter DF 40 Inches, and the Depth BE 60; what's the Content in Beer and Wine Gallons, and Malt Buſhels?



*For* BEER.

| | | |
|---|---|---|
| The Bottom Diameter is | 40 | Inches. |
| The Top Diameter is | 30 | |
| Their Rectangle is | 1200 | |
| | 3 | |
| Three Times the Rectangle or Product is | 3600 | |
| The Sq. of the Difference of the Diameters is | 100 | |
| The Sum is | 3700 | |
| The Depth is | 60 | |
| The Dividend is | 222000 | |

1077) 222000,00 (206,12, the Content in Beer Gallons and Parts.
00660836
66307
0138
00

*For*

10.36

4.5'
2.5
3.56

### For WINE.

882) 222000,000 (251,700 Content in Wine Gallons and Parts.
4560860
15106
600

### For MALT.

8212,41) 222000,0000 (27,03 Content in Malt Buſhels.

### RULE II.

Take the Area of the greater Baſe, the Area of the leſſer, and a Geometrical Mean between the two Areas; the Sum of theſe Three multiplied by one third Part of the Tun's Depth, gives the Content: And this is a general Rule for the Fruſtum of any Pyramid, whoſe Baſes are Parallel, whether they are ſquare, triangular, circular, *&c.*

### EXAMPLE.

Let the Tun be the ſame as above, whoſe Top Diameter is 30 Inches, the Bottom 40, and the Depth 60; what's the Content in Beer, Wine, and Malt?

| | |
|---|---|
| The Area of the Top Diameter (*viz.* 30) is | 2,5069 |
| The Area of the Bottom Diameter (*viz.* 40) is | 4,4568 |
| The Mean between the two Area's is | 3,3425 |
| Their Sum is | 10,3062 |
| This Sum, *viz.* 10,3062, muſt be multiplied by | 20 |
| The Content required in Beer Gall. and Parts | 206,1240 |

### *For* WINE.

| | |
|---|---|
| The Area of the Top Diameter (*viz.* 30) is | 3,0612 |
| The Area of the Bottom Diameter (*viz.* 40) is | 5,4421 |
| The Geometrical Mean between the 2 Areas is | 4,0815 |
| Their Sum is | 12,5848 |
| | 20 |
| Which multiply'd by $\frac{1}{3}$ of 60, *viz.* 20, gives the Content. | 251,696 |

### *For* MALT.

| | |
|---|---|
| The Area of the Top Diameter (*viz.* 30) is | 0,3287 |
| The Area of the Bottom Diameter (*ziz.* 40) is | 0,5845 |
| The Geometrical Mean between the 2 Areas is | 0,4384 |
| Their Sum is | 1,3516 |
| Which being multiply'd by $\frac{1}{3}$ of the Heighth, 60, *i. e.* 20, will give the Content in Buſhels and Parts. | 20<br>27,032 |

## *By the Sliding Rule.*

### *For* BEER.

Set 18,94 (the Beer Gauge Point for a Circle) on D, to 1 upon C; then againſt any Diameters on D, is the Area on C.

| | |
|---|---|
| Thus againſt 30 on D, you'll find its Area. | 2,5069 |
| And againſt 40 on D, its Area. | 4,4568 |
| Then (by *Prob.* 1. in the Uſe of the Square Root by the Rule before-going) find a Mean between the Areas. | 3,3425 |
| Their Sum is | 10,3062 |

Which by the Lines A and B, multiply by 20, and the Product will be 206,1240, as above.

*For*

*For WINE.*

Set 17,14 on D, to 1 on C; then againſt 30 on D, is 3,0612
And againſt 40 on D, you'll find 5,4421
The mean Proportion between theſe two Areas is 4,0815

Their Sum is 12,5848

Then ſet 1 upon B, to 12,5848 on A; and againſt 20 on B, you'll find 251,6960, as above; and the ſame muſt be obſerved for Malt, Reſpect had to the proper Gauge Point.

## To gauge the lower *Fruſtum* of a parabolick Conoid, cut Parallel to the Baſe.

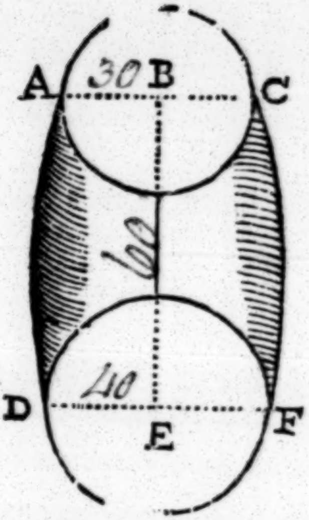
### *RULE.*

Add the Areas of the Diameters together, and multiply that Sum by Half the Heighth; ſo is the Product the Content of ſuch a Tun.

### *EXAMPLE* II.

Let ABCDEF repreſent the lower Fruſtum of a parabolick Conoid, whoſe Top Diameter is 30 Inches, the Bottom Diameter 40, and the Depth 60; what's the Content of the Tun in Beer and Wine Gallons, and Malt Buſhels?

For



*For* BEER.

Top Diameter 30
30

359) 900,0000 (2,5069 Area of the Top Diameter.
1825069
02542
232

Bottom Diameter 40
40

359) 1600,0000 (4,4568 Area of the Bottom Diameter.
1644568
20498
220

The Sum of the two Areas is 6,9637 which multiply'd by Half the Depth (*viz.* 60.) *i. e.* 30 gives the Content of the Tun, *viz.* 208,9110 in Beer Gallons and Parts.

*For*

### For WINE.

Top Diameter 30
30

294) 900,0000 (3,0612 Area of the Top Diameter.
0110662
18367
000

Bottom Diameter 40
40

294) 1600,0000 (5,4421 Area of the Bottom.
1304426
12652
002

The Sum is 8,5033 which multiply'd by Half the Depth, (*viz.* 60) *i. e.* 30 produceth the Content of the Tun in Wine Gallons and Parts.
255,0990

### For MALT.

Top Diameter 30
30

2737,47) 900,000000 (,3287 the Area of the Top.

The Bottom Diameter 40
40

2737,47) 1600,0000 (,5845 the Area of the Bottom.

The Sum of the Areas is ,9132, which multiply'd by Half the Depth (*viz.* 60) *i. e.* 30 shews the Content of the Tun to be 27,3960 Malt Bushels and Parts.

## *By the Sliding Rule.*

### *For* BEER.

Set 18,94 on D, to 1 upon C; then againſt 30 on D, is 2,5069, the Area of the Top Diameter on C; and as the Rule is now ſet, againſt 40 on D you'll find 4,4568 on C, the Area of the Bottom Diameter; then ſet 1 upon B, to 30 on A; and then againſt the Sum of the Areas on B, you'll find 208,9110 on A, the Content as above.

### *For* WINE.

Set 17,14 on D, to 1 on C; then againſt the Diameters, *viz.* 30 and 40 on D, you'll find 3,0612 and 5,4421 their Areas on C; then ſet 1 on B, to 30 on A, and againſt the Sum of the Areas, *viz.* 8,5033 on B, is 255,0990, the Content required on A.

### *For* MALT.

Set 52,32 on D, to 1 on C; then againſt 30 and 40 on D, you'll find ,3287 and ,5845 on C, their Areas; then ſet 1 on B, to 30 on A, and againſt ,9132 (the Sum of the Areas) on B, you'll find 27,3960 on A, as above.

### *RULE* II.

Multiply the Sum of the Squares of the Diameters by the Depth, and divide the Product by 718 for Beer Gallons, 588 for Wine, and 5474,94 for Malt Buſhels.

### *EXAMPLE.*

Let the Top Diameter be 20 Inches, the Bottom 30, and the Depth 40; what's the Content in Beer and Wine Gallons, and Malt Buſhels?

*For*

### *For* BEER.

Top Diameter 20  Bottom Diameter 30
20  30

Its Square 400  Its Square 900
400

The Sum of the Square is 1300
40

718) 52000,00 (72.42 Anſ.
17448(4
306(4
1(2

### *For* WINE.

588) 52000,00 (8843 Content in Wine Gallons and Parts.
496686
2501
23

Mult 7.497 ~~Mult 9.260~~

The ſame Dividend, being divided by 5474.94, will quote the Content in Buſhels; and the like for the Fruſtum of any other parabolick Conoid.

[illegible] 1771

## *To find the Content of any* Tun, *whoſe Sides are ſtraight, and Baſes Parallelograms or Ellipſes.*

### RULE.

1. To the greateſt Length *S t*, add Half the leſſer Length *o p*; then multiply the Sum by the greater Breadth *q r*, reſerving that Product.

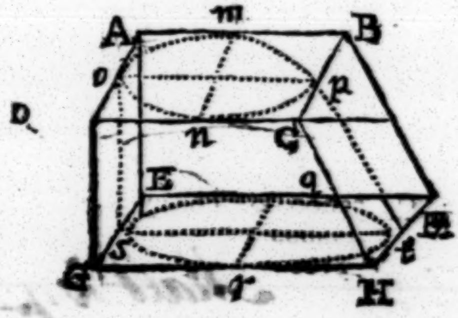
2. To the leſſer Length *o p*, add Half the greater Length *S t*; and multiply the Sum by the leſſer Breadth *m n*, add this Product to that before reſerved; then multiply the Sum by the Depth, and divide the Product

by

by 846 for Beer Gallons, and 693 for Wine, if the Baſes were Parallelograms; but by 1077 for Beer Gallons, 882 for Wine, and 8212,41 for Malt, if they were Ellipſes.

### *EXAMPLE.*

Let ABCDEFGH repreſent a Tun, whoſe Baſes are Parallel and Sides ſtraight, in which the ſtraight Lines at the Top and Bottom may repreſent a Tun, whoſe Baſes are Parallelograms, and the curved Lines may ſignify the Fruſtum of an elliptick Cone as below.



Let the Length of the greater Baſe of the Tun, *viz.* EF = *st* = GH be 60 Inches. and the Breadth of the ſame Baſe, *viz.* EG = *qr* = FH be 50 Inches; let the Length of the leſſer Baſe be DC = *op* = AB = 52 Inches, the Breadth of the ſame Baſe, *viz.* AD = *mn* = BC = 46, and the Depth DG = *os* = *&c.* 39; what's the Content in Beer, Wine, and Malt.

| | |
|---|---|
| The greater Length, *i. e.* EF = *&c.* is | 60 Inches. |
| Half the leſſer Length, *viz.* DC = *op*, *&c.* = 52. is | 26 |
| Their Sum is | 86 |
| Greater Breadth, *viz.* GE = *rq* = *&c.* is | 50 |
| The Product to be reſerved is | 4300 |

The

| | |
|---|---|
| The leſſer Length, *viz.* AB=*op*, *&c.* is | 52 |
| Half the greater Length, *viz.* EF=*&c.* is | 30 |
| Their Sum is | 82 |
| Leſſer Breadth AD=*mn*=*&c.* is | 46 |
| | 492 |
| | 328 |
| The Product is | 3772 |
| The Product above reſerved is | 4300 |
| Their Sum is | 8072 |
| The Depth is | 39 |
| | 72648 |
| | 24216 |

This Product, *viz.* 314808 is your Dividend. Now if your Figure has elliptical Baſes, your Diviſors are 1077 for Beer Gallons, 882 for Wine, and 8212,41 for Malt. But if the Baſes are Parallelograms, your Diviſors are 846 for Beer Gallons, 693 for Wine, and 6451,26 for Malt.

## To gauge the *Fruſtum* of an upright *elliptick Cone*, cut parallel to the Baſe another Way.

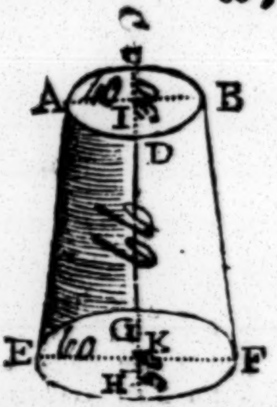
### *RULE.*

Multiply the Product of the Top Diameters by the Product of the Bottom Diameters, and extract the Square Root of that Product, which Root add to the Sum of the ſaid Products; multiply the laſt Sum by the Depth, then divde by 1077 for Beer Gallons, 882 for Wine, and 8212,41 for Malt Buſhels.

### *EXAMPLE.*

Let ABCDEFGHIK repreſent the Fruſtum of an upright elliptick Cone, whoſe Dimenſions are as below.

Greateſt Diameter at the Top AB is 40
Leaſt Diameter at the Top CD is 30
Greateſt Diameter at the Bottom EF 60
Leaſt Diameter at Bottom GH 50
Depth IK 66
} what's the Content in Beer, Wine, and Malt?



| | | |
|---|---|---|
| The greateſt Diameter at the Top is | | 40 |
| Leaſt Diameter at the Top is | | 30 |
| Their Rectangle or Product is | | 1200 |
| The greateſt Diameter at the Bottom is | | 60 |
| The leaſt Diameter at the Bottom is | | 50 |
| Their Rectangle or Product is | | 3000 |
| Rectangle of the Top Diameters is | 1200 | |
| Rectangle of the Bottom Diameters is | 3000 | |

Their Product is 3600000,0000 (1897,36 Root. Or  
Mean proportion

1

2) 260  
224

36) 3600  
3321

378) 27900  
26509

3794) 139100  
113829

37946) 2527100  
2276796

250304

Rectangle of the Top Diameters 1200  
Rectangle of the Bottom Diameters 3000  
A mean Proportional is 1897,36

Their Sum is 6097,36  
The Depth is 66

3658416  
3658416

The Dividend is 402425,76

*For* B E E R.

1077) 402425,76 (373,65 Content in Beer Gallons and Parts.  
07933451  
039087  
0753  
00

*For* WINE.

882) 402425,76 (456,26 the Content in Wine Gallons and Parts.
4962334
55374
354

*For* MALT.

8212,41) 402425,76 (49,00 Content in Buſhels and Parts.

2. Or (according to the general Rule before given for the Fruſtum of a Cone) you may find the Area of the greater Baſe, the Area of the leſſer Baſe, and a mean Proportional between them; and then multiply the Sum of theſe by $\frac{1}{3}$ of the Heighth, ſo will the Product be the Anſwer.

EXAMPLE.

Let the Figure and Dimenſions be the ſame as in the firſt Caſe; what's the Area in Beer, Wine, and Malt?

*For* BEER.

As 359 is to 30, ſo is 40 to 3,3426 the Area of the Top Diameter.
30

359) 1200,0000 (3,3426
1233426
15926
020

As 359 is to 50, ſo is 60 to 8,3565 the Area of the Bottom Diameter.
60

359) 3000,0000 (8,3565
1283565
20396
211

Area

| | |
|---|---|
| Area of the Top | 3,3426 |
| Area of the Bottom | 8,3565 |
| A Mean | 5,2851 |
| Their Sum is | 16,9842 |
| One third of the Heighth | 22 |
| | 339684 |
| | 339684 |
| Content of the Tun (near that above) is | 373,6524 |

### *For* WINE.

1. As 294 is to 30, ſo is 40 to 4,0816 Area of the Top Diameter.

2. As 294 is to 50, ſo is 60 to 10,2040 Area of the Bottom.

The Geometrical Mean is 6,4536

The Sum is 20,7392 which multiply by $\frac{1}{3}$ of the Heighth (*viz.* 66) *i.e.* 22 and the Product

414784
414784

will be 456,2624 the Cont. requir'd.

### *For* MALT.

1. As 2737,47 is to 30, ſo is 40 to ,4383, the Area at the Top.

2. As 2737,47 is to 50, ſo is 60 to 1,0959, the Area at the Bottom.

The Geometrical Mean is ,6903

The Sum is 2,2245 which multiply by $\frac{1}{3}$ of the Heighth (*viz.* 66) *i.e.* 22 and the Product will be 48,939 the Cont. requir'd.

*By*

## *By the Sliding Rule.*

### *For* BEER.

1. Set 359 on B, to 30 on A; then againſt 40 on B, is 3,3426 (the Area at the Top) on A.

2. Set 359 on B, to 50 on A; then againſt 60 on B, is 8,3565 (the Area of the Bottom) on A.

3. Then ſet 3,3426 on C, to 3,3426 on D; then againſt 8,3565 on C, is 5,2851 on D, for a mean Proportional; all which add to the two Areas before found, and their Sum is 16,9842; which (by the Rule) multiply'd by 22, gives 373,6, *&c.* the Content. on A

## To find the Content in Wine Gallons.

1. Set 294 on B, to 30 on A; then againſt 40 on B, is 4,0816 (the Area of the Top) on A.

2. Set 294 on B, to 50 on A; then againſt 60 on B, is 10,2040 (the Area of the Bottom) on A.

3. Set 4,0816 on the Line C, to 4,0816 on D; then againſt 10,2040 on C, is 6,4536 (a Geometrical Mean between the two Areas) on D, which add to the ſaid Areas, and the Sum will be 20,7392.

4. Set 1 upon B, to $\frac{1}{3}$ of the Heighth (*viz.* 66) 22 on A; then againſt 20,7392 on B, is 456,2624 (the Content required) on A.

### *For* MALT.

| | |
|---|---|
| 1. Set 2737,47 on B, to 30 on A; then againſt 40 on B, is the Area of the Top on A, *viz.* | ,4383 |
| 2. Set 2737,47 on B, to 50 on A; then againſt 60 on B, is the Area of the Bottom on A, *viz.* | 1,0959 |
| 3. Set ,4383 on C, to 4383 on D; then againſt 10,959 on C, is the Mean on D, *viz.* | ,6903 |
| The Sum is | 2,2245 |

4 Set 1 on B, to 2,2245 on A; then againſt ,22 on B, is 48,939, the Content of the Tun in Buſhels required on A: And the like may be done with a Tun, whoſe Baſes are Parallelograms, and its Sides ſtraight, only inſtead of the former Diviſors; you muſt make Uſe of 282 for Beer Gallons, 231 for Wine, and 2150 for Malt Buſhels.

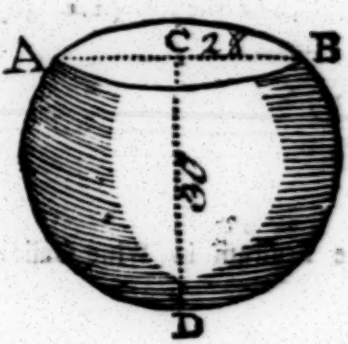
## To gauge the *Fruſtum* of a Globe.

### *RULE.*

To the Area of the Fruſtum's Diameter, add the Area of the Heighth, more one Third of the ſame; multiply the Sum by Half the Heighth, and the Product will be the Content of the Fruſtum.

### *EXAMPLE.*

Let ABCD repreſent the Fruſtum of a Globe, whoſe Diameter AB let be 28 Inches, and Altitude or Depth CD 30; what's the Content of the Fruſtum in Beer and Wine Gallons, and Malt Buſhels?



### *For* B E E R.

| | |
|---|---|
| 1. As 359 is to 28, ſo is 28 to the Area of the Diameter, *i.e.* | 2,1838 |
| 2. As 359 is to 30, ſo is 30 to the Area of the Depth, *i.e.* | 2,5069 |
| One third of the Area of the Heighth is | 0,8356 |
| Their Sum is | 5,5263 |
| Half the Altitude or Depth is | 15 |
| | 276315 |
| | 55263 |
| The Content of the Fruſtum is | 82,8945 |

### *For* W I N E.

| | |
|---|---|
| 1. As 294 is to 28, ſo is 28 to the Area of the Diameter, *i.e.* | 2,6666 |
| 2. As 294 is to 30, ſo is 30 to the Area of the Depth, *i.e.* | 3,0612 |
| One Third of the Area of the Alt. (*viz.* 30) is | 1,0204 |
| Their Sum is | 6,7482 |
| Half the Depth (30) is | 15 |
| | 337410 |
| | 67482 |
| The Cont. of the Fruſtum in Wine Gallons | 101,2230 |

### *For* MALT.

1. As 2737,47 is to 28, ſo is 28 to the Area of the Top ,286
2. As 2737,47 is to 30, ſo is 30 to the Area of the Depth, 328

One third of the Area of the Depth is ,109

Their Sum is ,723
Half the Depth is 15

3620
724

The Content in Buſhels 10,855

## *By the Sliding Rule.*

### *For* BEER.

Set 18,94 (the Beer Gauge Point for a Circle) on D, to 1 on C; then againſt the Diameter, *viz.* 28 on D, you'll find the Area thereof on C, *i. e.* 2,1838
And againſt the Depth, *viz.* 30 on D, you'll find 2,5069 on C.
The 3d of the laſt Area found, *viz.* of 2,5066 is 0,8356
Their Sum is 5,5263

Then ſet 1 upon B, to Half the Depth on A; and againſt the Sum, *viz.* 5,5256 on B, you'll find 82,8945 the Content required on A, *i. e.* 82 Gallons, 1 Pottle, 1 Quart, 1 Pint, and ,156 of a Pint.
,156

The Operation for Wine and Malt by the Sliding Rule, is in all Reſpects the ſame as above: only inſtead of 18,94, you muſt make uſe of 17,14 as a Gauge Point for Wine, and of 52,32 as a Gauge Point for Malt.

I have inſerted the preceding Rule for this Purpoſe, being expeditious enough both by the Pen and Inſtrument,

ment, nay, more ready and general than any other that I have taken Notice of. But if you'd have one more ſhort, only for the Pen, take the following Rule. To three Times the Square of the Fruſtum's Diameter, add four Times the Square of the Depth, multiply the Sum by the Depth, and divide the Product by 2154 for Beer Gallons, 1764 for Wine, and 1642,82 for Malt Buſhels.

## Of *Cask* Gauging.

Before we deliver Rules for this Purpoſe, 'twill not be amiſs to let the Learner know that the Requiſites are,

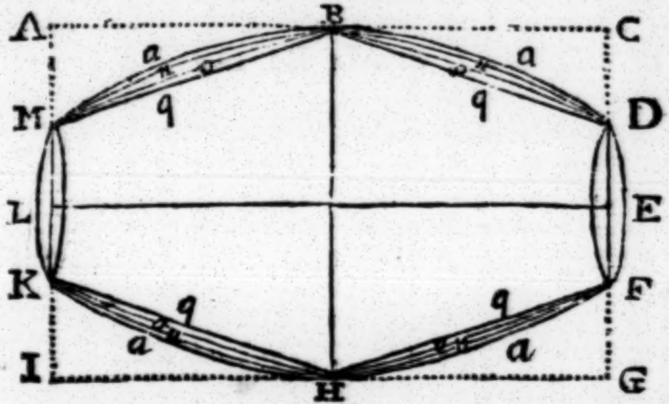
The Diameter at the Bung }
The Diameter at the Head } in Inches.
And the Length }

When theſe Dimenſions are known, our next Buſineſs is to conſider what the Nature or Form of the Cask we are to gauge is; for all Authors agree, that there are four Sorts, *viz.*

1. The middle Fruſtum of a Spheroid.
2. The middle Fruſtum of a parabolick Spindle.
3. The middle Fruſtum of two parabolick Conoids, abutting upon one common Baſe; or
4. The middle Fruſtum of two Cones, abutting upon one common Baſe.

To theſe we may add, that of a Cylinder, to which all may be reduced; and though when it has the ſame Length as any of the Figures above mentioned, and a Diameter the ſame with that of the Bung, the Content will be more than any of them, as may be ſeen by the next Figure following:

Yet when (by the Rules following) we think that we have reduced any given Cask to a Cylinder, 'tis ſuch a Cylinder, whoſe Length is the ſame with that of the Cask given: But its Diameter is ſuch, that it is greater than the Head, and leſs than the Bung's Diameter of the ſame. This will be made more plain, when we ſee the Figures, and conſult the Rules following.



1. In this Figure, ABCGHI, may repreſent a Cylinder, whoſe Diameter AI=CG is equal to the Bung's Diameter BH of the inſcribed Casks; and conſequently (as by the Figure) muſt contain more than any of them.

2. The middle Fruſtum of a Spheroid may be repreſented by the next inſcribed Cask, *viz.* M*a* B*a* DEF*a* H*a* KL.

3. The middle Fruſtum of a parabolical Spindle may be repreſented by the next inſcribed Cask, *viz.* M*n* B*n* DEF*n* H*n* KL.

4. The middle Fruſtum of two Conoids may be repreſented by the Cask M*o* B*o* DEF*o* H*o* KL.

5. The middle Fruſtum of two Cones (being the leaſt of all) may be repreſented by the Cask M*q* B*q* DE F*q* H*q* KL, now by this Figure 'tis plain, that a ſpheroidal Cask is the greateſt (except the Cylinder;) the middle Fruſtum of a Parabolick, the next leſs; the middle Fruſtum of two Conoids, leſs than that; and that of two Cones, leaſt of all, as before. This will yet appear more fully by their Contents as below.

## PROB. I.

*There's a Cask ſuppoſed to be the middle Fruſtum of a Spheroid, as* Ma Ba DEF*a* Ha KL, *whoſe*

| | | |
|---|---|---|
| Bung Diameter BH is | 40 | Inches. |
| The Head Diameter MK=DF is | 30 | |
| And the Length LE | 54 | |

How many Ale or Wine Gallons will the Cask contain?

### *RULE.*

To the Sum and half Sum of the Squares of the Bung and Head Diameters, add half the Difference of the ſaid Squares; the Sum of theſe multiplied by the Length, and the Product divided by 1077 for Beer Gallons, and 882 for Wine, quotes the Content reſpectively.

### *EXAMPLE.*

| | |
|---|---|
| The Squ. of BH the Bung's Diam. (*i. e.* 40) is | 1600 |
| The Sq. of MK=DF, the Head Diam. (*i.e.* 30) is | 900 |
| The Sum of theſe Squares is | 2500 |
| Half the Sum is | 1250 |
| Half the Difference of the Square is | 350 |
| Sum is | 4100 |
| Length is | 54 |
| | 164 |
| | 205 Ale. |

1077) 221400,00 (205,57
00600551
60161
0617
00

832)

Wine.

```
882) 221400,00 (251,02
       4500806
        09183
         010
```

| | |
|---|---|
| Content in Ale | 205,57 |
| Content in Wine | 251,02 |

## PROB. II.

*There's a Cask (of the ſame Diameters and Length with the former) ſuppoſed to be the middle Fruſtum of a parabolick Spindle, (as* M *n* B *n* DEF *n* H *n* KL,*) how many Gallons of Ale and Wine will this Cask contain?*

### RULE.

To the Sum and half Sum of the Squares of the Bung and Head Diameters, add three Tenth Parts of the Difference of the ſaid Squares; multiply the Sum by the Length, and divide the Product by 1077 for Beer Gallons, and 882 for Wine.

### EXAMPLE.

| | |
|---|---|
| The Sq. of BH, the Bung's Diam. (*i e.* 40) is | 1600 |
| The Sq. of MK=DF, the Head Diam. (*i.e.* 30) is | 900 |
| Their Sum is | 2500 |
| Half Sum is | 1250 |
| The Difference of the Sq. multiplied by ,3 is | 210 |
| The Sum of theſe is | 3960 |
| Length is | 54 |
| | 1584 |
| | 19,80 |

Ale.

1077) 21384000(198,55
10611455
092956
2551
00

882) 213840,00 (242,45 Wine.
3746622
21939
347

| | |
|---|---|
| The Content in Ale Gallons and Parts is | 198,55 |
| And in Wine | 242,45 *fere.* |

## PROB. III.

*There's a Cask whoſe Staves between the Bung and Head have but little Curvature, as* M *o* B *o* D E F *o* H *o* K L, *(in the laſt Figure,) and ſuppos'd to be the middle Fruſtum of two Conoids; what's the Area of this Cask in Ale and Wine Gallons, ſuppoſing it to have the ſame Diameters and Length with thoſe above?*

RULE

### *RULE.*

To the Sum and half Sum of the Square of the Bung and Head Diameters, add one Tenth of the Difference of the ſaid Squares; then multiply the Sum by the Length, and divide the Product by 1077 for Beer Gallons, and 882 for Wine.

### *EXAMPLE.*

| | |
|---|---|
| The Sq. of the Bung's Diam. *viz.* BH = 40 is | 1600 |
| The Sq. of the Head Diam. *viz.* AK = DF is | 900 |
| The Sum of the Squares is | 2500 |
| Half the Sum of the Squares is | 1250 |
| One Tenth of the Difference of the Squ. is | 70 |
| The Sum is | 3820 |
| The Length is | 54 |
| | 15280 |
| | 19100 |

Ale.

1077) 206280,00 (191,53
09855359
016741
0532

882) 206280,00 (233,87 Wine.
2982446
34786
766

| | |
|---|---|
| The Content in Ale Gallons and Parts is | 191,53 |
| And in Wine | 233,87 |

PROB.

## PROB. IV.

*There's a Cask in the Form of the middle Fruſtum of two Cones, abutting upon one common Baſe, as* M *q* B *q* DEF *q* H *q* KL, *having the ſame Diameters and Length as thoſe above; what's the Content in Beer and Wine Gallons?*

### RULE.

From the Sum and half Sum of the Squares of the Bung and Head Diameters, ſubſtract Half the Square of the Difference of the Diameters themſelves; then multiply the Remainder by the Length, and divide the Product by 1077 for Beer Gallons, and 882 for Wine.

### EXAMPLE.

| | |
|---|---|
| The Square of the Bung's Diam. is | 1600 |
| The Square of the Head Diam. is | 900 |
| Their Sum is | 2500 |
| Half the Sum is | 1250 |
| The Sum of theſe is | 3750 |
| Difference of the Diam is 10; which ſquared is 100. Half of which is | 50 |
| The Remainder is | 3700 |
| The Length is | 54 |
| | 148 |
| | 185 |

Ale.

1077) 199800,00 (185,51

09214553

059567

0515

00

882)

```
882) 199800,00 (226,53   Wine.
     2346804
      57675
        420
```

| | |
|---|---|
| The Content in Ale Gallons and Parts is | 185,51 |
| And in Wine | 226,35 |

By the four preceding Problems it appears, that if the Dimenſions are as in the firſt, *viz.* The

| | | |
|---|---|---|
| Bung's Diameter | .40 | Inches. |
| Head Diameter | 30 | |
| And Length | 54 | |

That the Content in each Variety will

| | | | | | |
|---|---|---|---|---|---|
| By *Prob.* | 1. | 205,57 | Ale Gall. and | 251,02 | Wine Gallons. |
| | 2. | 198,55 | | 242,44 | |
| | 3. | 191,53 | | 233,87 | |
| | 4. | 185,51 | | 226,53 | |

Where we may obſerve, that the Difference betwixt the firſt and ſecond, is the ſame as between the ſecond and third; and that of the third and fourth, but a little different from the other: By which it appears, that there's not ſo much Room left for an Error, as was by ſome Rules formerly preſcribed.

It can't be denied, but that a Cask having the ſame Diameters and Length as that above, may contain ſome certain Quantity between the Contents here found: However in ſuch Caſes you may proceed by the preceding Rules, adding 1, 2, 3, 4, or 5, tenth Parts of the Difference of the Squares (as Mr. *Everard* directs) according as your Judgment, and the Shape of the Cask ſhall direct.

This Caution muſt alſo be obſerved in finding a mean Diameter, when you would reduce a Cask to a Cylinder, in order to gauge it with the Sliding Rule.

### *To gauge the middle Fruſtum of a* Spheroid *another (and more expeditious) Way than is directed in the firſt preceding* Problem.

*RULE* I.

To twice the Square of the Bung's Diameter, add once the Square of the Head; multiply the Sum by the Length, and divide the Product by 1077 for Beer Gallons, and 882 for Wine.

*EXAMPLE.*

Let the Bung's Diameter be 34 Inches, the Head 24, and Length 48; what's the Content in Beer and Wine Gallons?

*For* BEER.

| | | | |
|---|---|---|---|
| Bung's Diameter is | 34 | Head Diameter | 24 |
| | 34 | | 24 |
| | 136 | | 96 |
| | 102 | | 48 |
| Twice its Square | 1156<br>1156 | its Square | 576 |
| Square of the Head | 576 | | |
| Their Sum is | 2888 | | |
| The Length is | 48 | | |
| | 23104 | | |
| | 11552 | | |

1077) 138624,00 (128,71 Ale Gallons.
030988 13
093643
0713
00

882)

882) 138624,00 (157,17 Wine Gallons.
5042086
63510
160

| | | |
|---|---|---|
| Area in Beer | 128,71 | Gallons. |
| Area in Wine | 157,17 | |

## *To gauge a Cask taken as the middle Fruſtum of a* parabolick Spindle *another Way.*

### *RULE.*

To twice the Square of the Bung's Diameter, add once the Square of the Head, from the Sum take $\frac{2}{5}$ of the Square of the Difference between the Bung and Head Diameters; multiply the Remainder by the Length, and divide the Product by 1077 for Beer Gallons, and by 882 for Wine.

### *EXAMPLE.*

Let the Dimenſions be the ſame as in the laſt Example of a ſpheroidal Cask; what's the Content in Beer and Wine Gallons?

| | |
|---|---|
| Twice the Square of the Bung's Diameter is | 2312 |
| Once the Square of the Head is | 576 |
| Their Sum is | 2888 |
| The Difference betwixt the Head and Bung Diameter is 10, its Square is 100; two Fifths of which is | 40 |
| Remainder is | 2848 |
| Length is | 48 |
| | 22784 |
| | 11392 |
| The Dividend is | 136704 |
| | 1077) |

```
1077)  136704,00  (126,93  Beer Gallons.
       02906279
        074023
         1030
          00

 882)  136704,00  (154,99  Wine Gallons.
       4850622
        44728
         882
```

When in Practice you would reduce any of the beforementioned Casks to a Cylinder, you may do it near enough the Truth by the Direction following.

### RULES.

Take the Difference of the Bung and Head Diameters of any Cask, and multiply that Difference by the Number which ſtands againſt the Name of the Cask given in the following Table: Add the Product to the Head Diameter, ſo will the Sum be the Diameter of a Cylinder; (which being of the ſame Length with the given Cask, will contain as much;) Square the Diameter thus found, and multiply that Square by the Length, and multiply that laſt Product by ,0028 for Beer Gallons, and ,0034 for Wine.

The Multipliers for a Cask, which is taken for *Varieties.*

|  |  |  |  |  |
|---|---|---|---|---|
| The middle Fruſtum of | 1 | a Spheroid | is | ,7 |
|  | 2 | a para. Spin. |  | ,63 |
|  | 3 | two Conoids |  | ,56 |
|  | 4 | two Cones. |  | ,51 |

### EXAMPLE.

Let a Cask be taken as the middle Fruſtum of a Spheroid, the Bung's Diameter let be 20 Inches, the Head 15, and Length 24; what's the Content in Beer and Wine Gallons according to this Rule?

| | |
|---|---|
| Bung's Diameter is | 20 Inches. |
| Head Diameter is | 15 |
| Difference of Diameters | 5 |
| The Cask's ſpheroidal, therefore the Mult. is | ,7 |
| The Product is | 3,5 |
| The Sum of the Pro. and Head Diameter is | 18,5 |
| | 18,5 |
| | 925 |
| | 1480 |
| | 185 |
| The Square of the Sum is | 342,25 |
| The Length is | 24 |
| | 136900 |
| | 68450 |
| Product is | 821400 |
| Multiplier for Beer | ,0028 |
| | 65712 |
| | 16428 |
| The Content in Beer Gallons | 22,9992 |
| The Multiplicand | 8214 |
| The Multiplier for Wine | ,0034 |
| | 32856 |
| | 24642 |
| The Content in Wine | 27,9276 Gallons |

More Examples of this Kind would be ſuperfluous; for what is ſaid of a Caſk taken as the middle Fruſtum of a Spheroid, may be underſtood of the other three, only inſtead of multiplying the Difference of Diameters by ,7 as above, you muſt uſe ,63, ,56, or ,51, according to the Denomination of the Caſk you are to gauge.

## To gauge Caſks by the *Sliding Rule.*

Upon the Side of the Sliding Rule is placed a Line of Inches, and under it the three firſt Varieties of Caſks, *viz.* The middle Fruſtum of a Spheroid, the middle Fruſtum of a Parabolick Spindle, and the middle Fruſtum of two Conoids, and on ſome Rules the middle Fruſtum of two Cones is inſerted upon the oppoſite Side to the others, and has there a Line of Inches fitted to it alſo: The Uſe of theſe Lines is to reduce a Caſk in any of the before-mentioned Forms to a Cylinder; for which purpoſe you may obſerve the Directions following.

Let a Caſk be given to be gauged in the Form of the middle Fruſtum of a Spheroid, having the ſame Dimenſions as above, *viz.*

*Inches.* Ans. 23 Gallns

The Bung 20 }
The Length 24 } what's the Content?
And Head Diameter 15 }

Firſt, Subſtract the Head Diameter from that of the Bung, and (in this Caſe) you'll find the Difference to be 5 Inches. Find this Difference in the Line of Inches, and againſt that (in the Line for the firſt Variety) you'll find 3, and almoſt 5, which you may call 3,5; add this 3,5 to the Head Diameter, *viz.* 15, and the Sum, *viz.* 18,5 will be the Diameter of an equal Cylinder.

Then ſet 18,94 (the Gauge Point for Cylinders) on D, to the Caſk's Length, *viz.* 24 upon C; then againſt the mean Diameters, *viz.* 18,5 on D, you'll find 22,99 on C, and

153.1

C. and that is the Content of the Cask in Beer Gallons required.

By this Example you may proceed to find the Content of a Cask in any other Denomination or Variety.

## *To find ſecond Gauge Points, when the firſt will not perform the Queſtion at one Operation.*

Here we muſt remember, that the Diviſors for a Parallelopipedon was 282 for Beer Gallons, and 231 for Wine, and 2150 for Malt Buſhels; and for a Cylinder, 359 for Beer Gallons, 294 for Wine, and 2737,47 for Malt, and that the Square Roots of theſe ſeveral Diviſors are as below; which may be inſerted upon the Line D, to uſe as Occaſion requires, and are commonly called Gauge-points.

| | |
|---|---|
| Diviſors for Squares, *&c.* | 282 for Beer Gallons. |
| | 231 for Wine Gall. |
| | 2150 for Malt Buſh. |

| | | |
|---|---|---|
| Gauge-points or Square Roots | 16,79 | Beer. |
| | 15,19 | Wine. |
| | 46,37 | Malt. |

| | |
|---|---|
| Diviſors for Circles, *&c.* | 359 for Beer Gallons. |
| | 294 for Wine Gallons. |
| | 2737,47 for Malt Buſh. or 2738 |

| | | |
|---|---|---|
| Gauge-points or Square Roots | 18,94 | Beer. 95 |
| | 17,14 | Wine 15 |
| | 52,32 | Malt. |

The Uſe of theſe Points, and others alſo, (which here I omit,) are ſufficiently taught in the preceding Work. Where you may obſerve, when any of them is uſed in gauging Solids, that you are always to ſet that Gauge-point, which according to the Nature of the Figure you are to uſe, on the Line D, to the Length or Depth of the

282
282
564
2256
564
79524

48 384
562
282

1895
1895
9475
17055
15160
1895
3591425

the Solid, on the Line C; and then againſt the Diameter or Side thereof on D, you have its Content on C. But in ſome Caſes, the Content by theſe Gauge Points can't be found at one Operation: as for Example, ſuppoſe the Diameters of a Cylinder to be 80 Inches, and Depth 76, and the Content in Beer Gallons be required; in this Caſe, when I ſet 18,94 on D, to 76 the Depth on C, I find 80 the Diameter will fall upon no Part of the Line C; which ſhews, that a Cylinder having theſe Dimenſions, can't conveniently be gauged by this Gauge-point at one Set of the
q· turn Rule, without taking Half the Length, *&c.* And though this may ſerve Turn well enough, yet 'tis neither ſo expeditious nor artificial as when done at one Set of the Rule, without any ſuch halving and doubling as hitherto has been uſed among moſt Officers and Others.

## CASE I.

When the Lines C and D are ſet even at both Ends, if you find any Gauge-point on D ſtand on any Part of the firſt *Radius* of the Line C; the ſecond Gauge-point (to that) on D, will fall againſt ſome Part of the ſecond *Radius* of the Line C; but to know on what Part of the Line D, the ſecond Gauge-point will fall, take the following

### *RULE.*

Multiply the Square of any ſuch Gauge-point by 40, and divide the Product by 4; then is the Square Root of the Quotient the ſecond Gauge-point required.

### *EXAMPLE.*

Suppoſe I would find a ſecond Gauge-point to 18,94, in order to ſolve the preceding (or any ſuch) Queſtion,

where the firſt proves deficient, the Square of this firſt
Gauge-point is 359,05
Which multiply by 40

and divide the Product by 4,) 14362,00 (3590,50
230200

The Quotient is 3590,5, whoſe Square Root is 59,92, at which you may make a Point or Prick on the Line D, for a ſecond Gauge point to 18,94; or in this Caſe, inſtead of multiplying the Square of the Gauge-point by 40, and dividing by 4, you may only move the decimal Point in the Square, one Place farther towards the Right-Hand; ſo will 359,05 become 3590,5 as before. Now to anſwer the former Problem by the ſecond Gauge-point, you may proceed as before, *i. e.* ſet 59,92 on the Line D, to the Depth of the Cylinder, *viz.* 76 on C; then againſt 80 the Diameter on D, you'll find the Content required, *viz.* 1354,8764 Beer Gallons on C. By the ſame Rule you'll find the ſecond Gauge point to 17,14 for Wine, to be 54,22; at which you may alſo make a Point or Prick on the Line D, to uſe in ſuch Caſes; where the firſt is deficient, and ſo of any other Gauge-point (in the Caſe) whether for a Circle or Square.

## CASE II.

When the Lines C and D are ſet even at both Ends, if you find the firſt Gauge-point on the Line D, to ſtand againſt any Part of the ſecond *Radius* of the Line C; the ſecond Gauge-point on D, will fall againſt ſome Part of the firſt *Radius* of the Line C; but to find upon what Place of the Line D, ſuch a Point will fall, you may take the following

### *RULE.*

Multiply the Square of any ſuch (1.) Gauge-point by 4, and divide the Product by 40; (or by 10, and divide by 100;) then the Square Root of the Quotient will be the ſecond Gauge-point required.

*EXAMPLE.*

Let 52,32 be the circular Gauge-point for Malt Buſhels, and let a Cylinder, whoſe Diameter is 114 Inches, and Depth 20, be given to be gauged, *i. e.* the Content in Malt Buſhels is required.

Set 52,32 on D, to 20 the Depth on C; then you'll find that 114 the Diameter on D, will fall on no Part of the Line C; which ſhews, that the Cylinder can't conveniently be gauged at one Set of the Rule by this Gauge-point; therefore another muſt be found, as below.

The Square of the Gauge-point, *viz.* 52,32 is 2737,3824
Which multiply by 4,
And divide by 40,) 10949,52960 (273,73824
294952960
1213010

The Quotient is 273,73824, whoſe Square Root is 16,545 for the ſecond Gauge-point required.

Now to gauge the Cylinder, ſet 16,545 on D, to 20 the Depth on C; then againſt 114, the Diameter on D, you'll find Malt Buſhels on C, which (by this ſecond Gauge point) is the Content at one Set. In this Caſe, the Square of the ſecond Gauge point might have been found, if (inſtead of multiplying by 4, and dividing by 40) you had placed the decimal Point, one Place towards the Left-Hand in the Square of the firſt Gauge-point given, ſo would 2737,47 become 273.747.

If you would find the ſecond Gauge-points by the Inſtrument it ſelf, you may ſet any Number upon the firſt or ſecond *Radius* of the Line C, to the firſt Gauge-point on the Line D; then againſt the ſame Number made ten Times leſs, or ten Times more; on the *Radius* of C, will ſtand a Number on D, which is the Gauge point required, and muſt be valued according to that Side of the firſt Gauge-point it falls on, *i. e.* if on the left Side leſs, but if on the right Side of the firſt Gauge-point it will be more than it: As if it were required (inſtrumentally) to find a ſecond Gauge-point to 18,94, I ſet 4 on the firſt *Radius* of the Line C, to 18,94 on D; then againſt 40 on the ſecond *Radius* of the Line C, I find 59,92 on the Line D, for the Gauge-point required. *Where note*, that if you had ſet 1, 2, 3, 4, or 5, *&c.* of the firſt *Radius* of the Line C, to 18,94 on D; then would 10, 20, 30, 40, or 50, *&c.* on the ſecond *Radius* of the Line C, fall on the ſame Place of the Line D, *viz.* on 59,92, as before.

## To find the Ullage of a *Cask*.

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Let the Figure following represent a Cask lying with its Axis parallel to the Horizon, partly empty: Let the Bung's Diameter (*viz.* AB) be 38 Inches, and the Content 130 Gallons: Let CB be the wet Inches=25, and AC the dry Inches=13; I demand how many Gallons are in the Cask, and how many are drawn out.

For the Solution of this and ſuch like Problems, I have here inſerted the following Table, from the 94th Page of Mr. *Ward's Gauger's Practice*; whoſe Uſe and Conſtruction is as below.

### *RULE.*

Divide either the wet or dry Inches by the Bung's Diameter, and the Quotient ſeek in the Table under V, the verſed Sine; againſt it ſtands a Number, which multiplied by the Content of the Cask, exhibits the Vacuity, if your Dividend were the dry; or remaining Liquor, if it were the wet Inches.

*EX-*

### EXAMPLE I.

The wet Inches are 25, which divide by

38) 25,00 (,65
220
3

the Quotient is ,65; and becauſe the Remainder, *viz.* 30, is ſo near the Diviſors, *viz.* 38, you may make the Quotient 66; which find under V, and againſt it (under the Word Area) you'll find ,7002; which multiplied by the Cask's Content (*viz.* 130) produceth 91,0260, the remaining Liquor required.

### EXAMPLE II.

The dry Inches are 13, which divide by

38) 13,00 (,34 Quotient.
168
0

Find this Quotient under V, and againſt it you'll find ,2998; which multiplied by the Cask's Content, *viz.* 130, produceth 38,9740, the Vacuity required, which added to the remaining Liquor, *viz.* 91,0260, makes 130, the Content, which may prove the Work to be true,

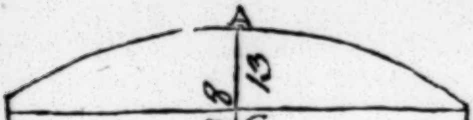
A TA-

## A TABLE of the Segments of a Circle.

| V | Area | V | Area | V | Area | V | Area |
|---|---|---|---|---|---|---|---|
| 1 | .0017 | 99 | .9983 | 26 | .2066 | 74 | 7934 |
| 2 | 0048 | 98 | .9952 | 27 | .2178 | 73 | .7822 |
| 3 | .0087 | 97 | .9913 | 28 | .2292 | 72 | .7708 |
| 4 | .0134 | 96 | .9866 | 29 | .2407 | 71 | .7593 |
| 5 | .0187 | 95 | .9813 | 30 | 2523 | 70 | .7477 |
| 6 | .0245 | 94 | .9755 | 31 | 2640 | 69 | .7360 |
| 7 | 0308 | 93 | .9692 | 32 | .2759 | 68 | .7241 |
| 8 | .0375 | 92 | .9625 | 33 | .2878 | 67 | 7122 |
| 9 | .0446 | 91 | .9554 | 34 | .2998 | 66 | 7002 |
| 10 | .0520 | 90 | .9480 | 35 | .3119 | 65 | .6881 |
| 11 | .0598 | 89 | .9402 | 36 | .3241 | 64 | 6759 |
| 12 | .0680 | 88 | .9320 | 37 | .3364 | 63 | 6636 |
| 13 | .0764 | 87 | .9236 | 38 | .3487 | 62 | 6513 |
| 14 | .0851 | 86 | 9149 | 39 | .3611 | 61 | .6389 |
| 15 | .0941 | 85 | .9059 | 40 | .3735 | 60 | .6265 |
| 16 | .1033 | 84 | .8967 | 41 | .3860 | 59 | 6140 |
| 17 | .1127 | 83 | 8873 | 42 | .3986 | 58 | 6014 |
| 18 | .1224 | 82 | .8776 | 43 | 4112 | 57 | .5888 |
| 19 | .1323 | 81 | .8677 | 44 | 4238 | 56 | .5762 |
| 20 | .1424 | 80 | .8576 | 45 | .4364 | 55 | 5636 |
| 21 | .1527 | 79 | .8474 | 46 | 4491 | 54 | .5509 |
| 22 | .1637 | 78 | 8369 | 47 | .4618 | 53 | 5382 |
| 23 | .1737 | 77 | .8263 | 48 | .4745 | 52 | .5255 |
| 24 | 1845 | 76 | .8155 | 49 | .4873 | 51 | .5127 |
| 25 | .1955 | 75 | .8045 | 50 | .5000 | 50 | .5000 |

Here the Reader may note, That thoſe Figures in this Table, which differ from thoſe in Mr. *Ward*'s Book abovementioned, are placed in the Room of thoſe there which I ſuppoſe were printed falſe; [illegible] againſt the verſed Sine 14 (in that Author) is ,0151 for the Area, but ought to be .0851, as in this Table, *&c.*

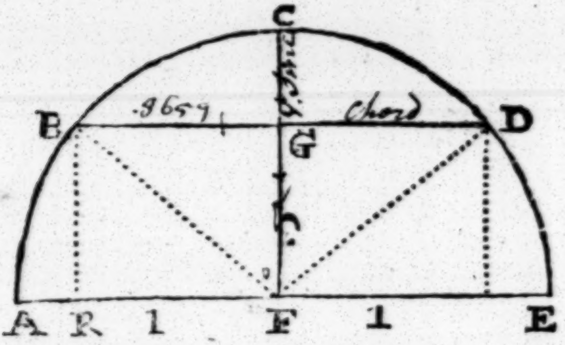
Cont.



Cont. 130.

## *The Conſtruction of the preceding Table.*

In the following Figure, that Part of it which is incloſed by the right Line BD, and Arch BCD, is called the Segment of a Circle; in which GC is called the verſed Sine, and BD its Chord: Alſo that Part of it which is incloſed by the *Radius* FB, FD, and Arch BCD, is called the Sector of a Circle, and AE the Diameter, which let be 2.



Having the Diameter AE=2, you may find the Circumference. Thus, as 1 : 3,14159265 : : 2 : 6,28318530, 8530, and this 6,28318530 is the Circumference of a Circle, whoſe Diameter is 2; the Area of which Circle (or any other) may be found by multiplying half the Circumference, (*viz.* half 6,28318530, *i. e.* 3,14159265) by half the Diameter, (*viz.* 2, *i. e* 1,) and the Product, (*viz.* 3,14159265) will be the Area; from whence 'tis plain, that the Length of half the Arch of any Sector of a Circle, is equal to the Area of that Sector.

When the Area of the Sector (FBCD is found,) and alſo the Area of the Triangle BGDF, their Difference is the Area of the Segment BCDG.

## EXAMPLE.

Let it be required to find the Area of the Segment BCDG, whoſe verſed Sine GC is 5, *i. e.* half the *Radius* FC=FE=FA=1.

*Firſt*, in order to find the Area of the Sector, we muſt find the Quantity of the Angle at the Center, *viz.* BFC=DFC; to find which we have FG=,5, and BG=,8659, (*i. e*) a mean Proportional between AF=1, +FC=5, (*i.e.* 1,5.) Thus,

| | |
|---|---|
| As FG (the Side of the right angled Triangle FBG right angled at G) that is, ,5 or ,5000. | 3,698970 |
| Is to Radius | 10,000000 |
| So is BG=,8659 | 3,937468 |
| To the Tangent of 59,99, *viz.* | 10,238490 |

By which it appears, that the Angle BFC in Degrees and decimal Parts is 59,99.

Now to find the Length of any Number of Degrees, (*i. e.* the Length of any Arch) of this Circle, ſay,

As 360 the Circumference in Degrees, is to 6,28318530, the Length of the Circumference in Inches and decimal Parts: So is 1 Degree to ,01745329, the Length of one Degree. Now the Length of any other Arch may be found thus:

As 1 Degree is to ,01745329 the Length thereof, ſo is any other Number of Degrees to the Length of their reſpective Arches; ſo that ,01745329 is a common Multiplicator for any Number of Degrees; ſo the Degrees of the Arch BC, *viz.* 59,99 being multiplied by ,01745329, gives 1,0470228671 the Length of the Arch BC; which being half the Length of the Arch BCD, muſt be the Area of the Sector FBCD, which was required.

## To find the Area of the Triangle BGDF.

In the Figure, there's given FG=,5, and BG=,8659, being the Breadth and Length of the Parallelogram RBCF (which is equal to the two Triangles TGB, and TGD) to

| | |
|---|---|
| find its Content: Thus BG is | ,8659 |
| And FG is | ,5 |
| The Area of the Parallelogram RBGF, or Triangle BFD is | ,43295 |
| From the Area of the Sector FBCD | 1,0470228671 |
| Take the Area of the Triangle BFD | ,43295 |
| And there will remain the Area of the Segment BCDG. | ,6140728671 |

But becauſe in all Tables that are for this purpoſe, 'tis ſuppoſed, that the Area of the whole Circle is Unity, the preceding Table was made by this Proportion.

As the Area of the Circle, whoſe Diameter is 2, *viz.* 3,14159265 is to the Segment of any Part of that Diameter,

So is 1 (the Area of another Circle) to the Segment of the like Part of its Diamerer: That is,

As 3,14159265 is to ,6140728671, (the Segment laſt found:) So is 1 to ,1954, the Segment anſwering to the verſed Sine 25 in the Table.

For as the verſed Sine of the firſt Segment, *viz.* GC was 5, *i. e.* $\frac{1}{2}$ the *Radius* FC; ſo is 25 the verſed Sine of the laſt Segment, $\frac{1}{2}$ of ,50 the *Radius* of the Table:

And what is here ſaid may ſuffice, both for the Uſe and Conſtruction of the Table, which may be uſed when the Caſk is cylindrical; but when the Bung and Head differ much, then the Line of Segments on the Sliding Rule is moſt convenient.

## *To calculate any Diameter between the Bung and Head of a ſpheroidal Caſk, (when the Axis is perpendicular to the Horizon) whoſe Dimenſions*

| | | | |
|---|---|---|---|
| are | AB the Length | 30 | Inches. |
| | CD the Bung's Diameter | 26 | |
| | EF the Head's Diameter | 22 | |

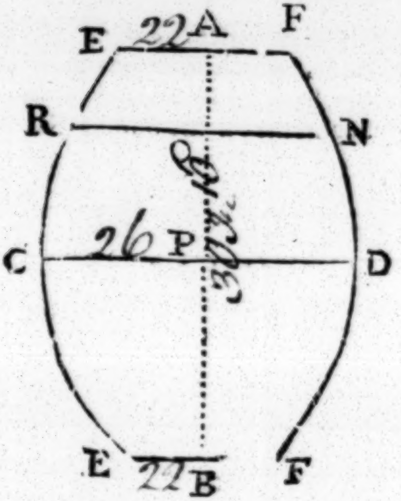
### *RULE.*

From the Square of the Bung's Diameter (CD,) ſubſtract the Square of the Head Diameter (EF,) and divide the Square Root of the Remainder by half the Caſk's Length (AB;) then multiply this Quotient by the Diſtance of the required Diameter from the Bung, which Product you may call your Subducend.

Laſtly, From the Square of the Bung's Diameter, ſubſtract the Square of your Subducend, and the Remainder is the Square of the Diameter required.

### *EXAMPLE.*

Let it be reqired to find the Diameter RN, which we'll ſuppoſe to be 10 Inches diſtant from CD, the Bung Diameter.



| | |
|---|---|
| The Square of the Diameter CD is | 676 |
| The Square of the Head Diameter EF is | 484 |
| The Difference of the Squares is | 192 |
| The Square Root of the Remainder (*viz.* 192) is | 13,85 |

This Root, *viz.* 13,92, divided by half the Cask's Length, *viz.* 15, quotes ,92; which multiplied by 10, produceth 9,2 for the Subducend.

| | |
|---|---|
| Square of the Bung-Diameter | 676 |
| Square of the Subducend is | 84,64 |
| There remains | 591,36 |

Which is the Square of the Diameter RN, whoſe ſquare Root is 24,31, the Diameter RN it ſelf: Having thus found the Diameter RN, you may find the Content of RNCDEF by the following

## RULE.

1. To twice the Square of the Bung-Diameter, add once the Square of the Diameter at the Liquor's Surface,

(which we ſuppoſe to be RN.) and multiply the Sum by the Diſtance from the Bung-Diameter to the Liquor's Surface.

2. To twice the Square of the Bung's Diameter, add once the Square of the Head; multiply the Sum by the Diſtance from the Bung Diameter to the Head, add theſe two Sums together, and divide their Sum by 1077 for Beer Gallons, and 882 for Wine

*EXAMPLE.*

| | |
|---|---|
| The Square of the Bung's Diameter | 676 |
| | 676 |
| Twice the Square of the Bung's Diameter is | 1352 |
| The Sq. of the Diam. at the Liquor's Surface, *viz.* RN | 591,36 |
| Their Sum | 1943,36 |
| Which multiply by 10, the Diſtance of RN from CD | 10 |
| The Product is | 19433,60 |
| Twice the Square of the Bung's Diameter is | 1352 |
| The Square of the Head is | 484 |
| Their Sum is | 1836 |
| Which Sum multiplied by Half the Length, *i.e.* | 15 |
| | 9180 |
| | 1836 |
| Produceth | 27540 |
| To which add the Product before found, *viz.* | 19433,60 |
| Their Sum is | 46973,60 |

Which Sum you may divide by 1077 for Beer Gallons, and 882 for Wine, as above directed; and 8212,41 for Malt, (if it be a Cask cut off parallel to the Bung-Diameter.)

*For* BEER.

1077) 46973,60 (43,61 the remaining Liquor in Beer Gallons.
038924(3
0666(6
01(5
(0

*For* WINE.

882) 46973,60 (53,25 the Content in Wine Gallons and Parts.
28772(0
221(1
5(7

Now having by the fore-going Rules found the Quantity of the remaining Liquor in Beer Gallons to be 43,61 you may take it out of the Cask's whole Content, *viz.* 51, 14 Gallons, and there will remain 7,5 ; which may inform you, that the Liquor drawn out is 7,53 Gallons, or it will want ſo much to fill it up: Where you may note, that if the Liquor's Surface had been below the Bung, the Difference of the Sums would have been your Dividend.

### *A more ſpeedy Way to find the remaining Liquor or Vacuity of a ſpheroidal Cask, poſited with its Axis perpendicular to the Horizon, or ſtanding at one End.*

*RULE.*

Divide the wet or dry Inches by the Cask's Length, and if the Quotient exceeds ,5000, add to the ſaid Quotient one tenth Part of the Exceſs: But if it be under ,5000, ſubſtract one tenth Part of the Want; ſo will the Sum or Remainder be a decimal Number; by which, if you multiply the Content of the Veſſel, the Product will be the Quantity of Liquor therein, if your Dividend was the wet Inches; but the Quantity of Liquor which is required to fill it up, if the Dividend was the dry Inches: But obſerve, that the decimal Places in the Dividend, muſt exceed thoſe in the Diviſor (if any be) by 4 Places, and the ſame Caution muſt be obſerved in the next Rule.

### *EXAMPLE.*

Bung-Diameter 28 } Ale Gall. } Wet Inches 18 }
Head 22 } Content 60,96 }
Length 32 } } Dry Inches 14 }

Wet Inc. Quot.
Length 32) 18,0000 (,5625 this exceeds, ,5000 by 0625,
,20860 ,00625 one tenth of which is 00625
010 ———
The Sum is ,56875 a Multiplicand, or, *&c.*
The Cont. of the Cask is 60,96
———
341250
511875
341250
——————
Liqu. in the Cask 34,6710000

### *To find the Vacuity or Quantity of Liquor that the Cask wants to fill it up.*

### EXAMPLE I.

Dry Inches.

The Length 32) 14,0000 (,4375 this wants ,0625 of 5000,
12460 ,00625 is one Tenth of ,0625.
210

This Remainder ,43125 the Multiplier, or, *&c.*
The Casks, whoſe Cont. is 60,96

258750
388125
258750

There want's to fill it up 26,2890000, to which add the Liquor in the Cask 34,6710000, and the Sum will

be the Cont. of the Cask 60,9600000 when full; which may ſerve as a Proof to both Operations.

### EXAMPLE II.

Length 32
Wet Inches 28
Dry 4
What Quantity of Liquor is in the Cask, and how much will fill it up.

Wet Inches.

Length 32) 280000 (,8750 this exceeds ,5000 by ,3750
24600 ,0375 is one Tenth of 3750
100

Sum is ,9125 for a Multiplier, or, *&c.*
Content 60,96

54750
82125
54750

The Gallons are 55,626000 which are now in the Cask.

Dry Inches.
Length 32) 4,0000 (,1250 this wants ,3750 of ,5000
0360 ,0375 is one Tenth of ,3750
100 ———
The Diff. Rem. is ,0875 for a Multiplier, or, &c.
The Cask's Content is 60 96

5250
7875
5250

wants to fill the Cask 5,334000
there is in the Cask 55,626000

their Sum is the Cont. 60,960000 for Proof.

### *To find the Ullage of a Cask which lies on one Side, or with its Axis parallel to the Horizon.*

#### *RULE.*

Divide the wet or dry Inches by the Bung's Diameter; if the Quotient be above ,5000, add to it one fourth Part of the Exceſs, and multiply the Sum by the Cask's whole Content. But if it be under ,5000, take from it one fourth Part of what it wants, and the Remainder will be a decimal Fraction, which multiplied into the Content, exhibits the Thing required.

#### *EXAMPLE.*

Let the Cask be the ſame as before, only in another Poſition.

| | | | |
|---|---|---|---|
| Bung-Diameter 28 | Ale Gall. | Wet 21 | Inch. |
| Head 22 | Content 60,96 | Dry 7 | |
| Length 32 | | what's in the Cask, and what will fill it up. | |

Wet Inches.

Bung 28) 21,0000 (,7500 this exceeds ,5000 by ,2500
14000 ,0625 is one fourth Part of ,2500
000

The Sum is — — — ,8125 for a Multiplier, or, &c.
The Cont. of the Cask is 60,96

48750
[illegible]
73125
48750

the Li. in the Cask is 49,530000

*Note*, That if the Quotient proves under ,5000, or above ,8000, inſtead of one fourth Part, add or ſubſtract one fifth Part.

## *To find the Vacuity, or Quantity of Liquor that the Cask wants to fill it.*

Dry Inches.

Bung 28) 7,0000 (,2500 this wants ,2500 of ,5000
14000 ,0625 is one fourth Part of ,2500
000

The Difference is ,1875 for a Multiplicator, or, &c.
the Cask's Content is 60 96

11250
16875
11250

There wants to fill the Cask } 11,430000. To which add
the contained Liq. 49,530000

the Cask's Content 60,960000

If the Cask you are to ullage be a Cylinder, or near that Form, the preceding Rule will be very near the Truth; but if the Bung-Diameter be much greater than the Head Diameter, then inſtead of one fourth Part (as above) you muſt add or ſubſtract but one third Part, and multiply the Sum or Remainder by the Content; ſo will the Product be what is required.

### *EXAMPLE.*

Let the Content of a Cask that bulges much, be 60 Gallons.

The Bung-Diameter 26 }
Wet 10 } Inches.
Dry 16 }

To find the Vacuity.

```
              Dry Inches.
Bung 26 )  16,0000    (,6153 exceeds ,5000 by ,1153
            04402      ,0384 is ⅓ of ,1153
              112     ────────
              Sum      ,6537
          Content         60
                      ────────
          Vacuity    39,2220 Gallons.
```

## *To ullage a Cask by the Sliding Rule, when its Axis is parallel to the Horizon.*

Let a Cask be in the Form of a midddle Fruſtum of a Spheroid.

```
                 Inches.
Whoſe Bung is    30 }                  Gall. { Wet 18 } Inches.
Head             22 } Content  80,58         { Dry 12 }
Length           38 }                        { what's the Vacui-
                                             { ty, and contained
                                             { Liquor.
```

To anſwer this Problem, or any other of the like Nature, you have a Line of Segments upon the ſame Side of the Rule with the Lines A and B, which Line is noted at the right End with Seg. *ly*, which may ſignify a Line of Segments for a lying Cask. To this Line belongs a Line of Numbers marked at the right End with Number. The Uſe of both are as below.

1. Set the Bung's Diameter, *viz.* 30 upon the Line of Numbers, to 100 on the Line of Segments; then againſt the wet Inches, *viz.* 18, on the Number, you'll find 63,9 on the Line of Segments, which keep.

2. Set 100 upon the Line B, to the Cask's Content, *viz.* 80,58 on A; then againſt the Number laſt found, *viz.* 63,9 on B, you'll find 51,49 Gallons; and ſo many Gallons are now in the Cask, becauſe you made Uſe of the wet Inches. But to know how many it wants of being full,

1. Set the Bung's Diameter, *viz.* 30 on the Line of Numbers, to 100 on the Line of Segments; then againſt the dry Inches, *viz.* 12 on the Line of Number, you'll find 36,1 on the Line of Segments, which keep.

2. Set 100 on the Line B, to the Cask's Content, *viz.* 80,58 on A; then againſt the Number laſt found, *viz.* 36,1 on B, you'll find 29,09 Gallons on the Line A, and ſo much is the Vacuity, becauſe you uſed the dry Inches.

To prove which

| | |
|---|---|
| The Vacuity is | 29,00 |
| The contained Liquor is | 51,49 |
| Their Sum is the Content | 80,58 |

and according to theſe two Examples you may proceed to ullage any other lying ſpheroidal Cask.

## *To ullage a ſpheroidal Cask, when its Axis is perpendicular to the Horizon, or ſtanding on one End, by the Sliding Rule.*

For this Purpoſe you have a Line of Segments for a ſtanding Cask on the ſame Side of the Rule with the Lines C and D, (if the Line of Cubes herein before deſcribed is not there,) noted with Seg. *St* againſt which is placed a Line of Numbers, both uſe as below.

Let the Cask be the ſame as in the laſt, only in another Poſition.

Inches.
Bung 30 }
Head 22 } Cont. Gall. 80,58 { Wet Inches 23 }
Length 38 } { Dry Inches 15 } what's the Vacuity and contained Liquor.

1. For the contained Liquor, ſet the Cask's Length, *viz.* 38 on the Line of Numbers, to 100 on the Segments; then againſt 23 the Wet Inches on the Line of Numbers, you'll find 61,5 on the Line of Segments, which keep.

2. Set 100 on B, to the Cask's Content, *viz.* 80,58 on A; then againſt the Number laſt found, *viz.* 61,5 on B, you'll find 49,57 on A, and ſo much Liquor is now in the Cask.

For the Vacuity, or the Liquor wanting to fill the Cask.

1. Set the Cask's Length, *viz.* 38 on the Numbers, to 100 on the Segments; then againſt 15 the dry Inches on the Numbers, you'll find 38,2 on the Segments, which keep.

2. Set 100 upon B, to the Cask's Content, *viz.* 80,58 on A; then againſt 38,2, the Number laſt found on B, you'll find 31,01 the Quantity of Liquor required to fill the Cask.

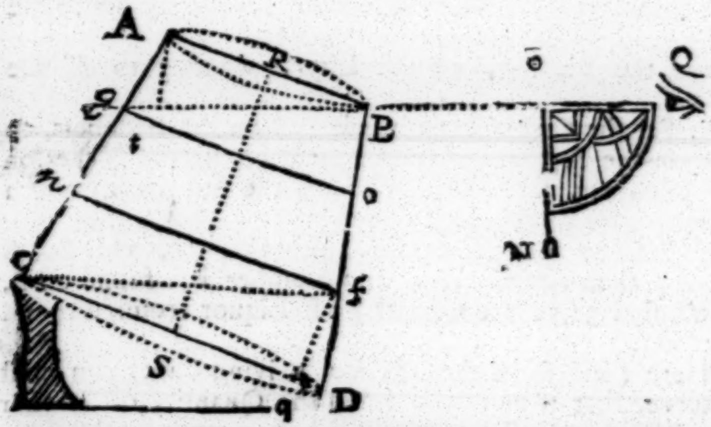
Here (and ſo in the laſt, *i. e.* a lying Cask) you muſt obſerve, that if you would find the Quantity of Liquor that is in any Veſſel of this Kind, you muſt ſet the Bung or Length to the wet Inches; But if the Vacuity, you muſt ſet them to the dry, according as the Cask is poſited.

## *To find the Drip or Fall of a Tun.*

Suppoſe a conical Tun (as ABCD) to be ſo poſited, as to have its Axis RS inclining towards the Horizon, ſo that when the Liqour juſt touches the Angle at C, (made by the bottom Diameter CD, and Side of the Cone AC,) it may wet 20,14 Inches of the oppoſite Side BD, *viz.* D*f*, or when filled ſo as to touch B, may want 18 Inches (on the oppoſite Side) of the Top as A*g*; let the Dimenſions of the Tun be firſt,

| | | |
|---|---|---|
| The Top Diameter AB | 90, | Inches. |
| Bottom Diameter CD | 106, | |
| Depth RS | 60, | |
| Slanting-ſide AC or BD | 60,53 | |

According to theſe Dimenſions, the Content of the Tun in Beer Gallons and Parts is 1608,69 Gallons.



When this Tun is (for Conveniency of cleanſing) ſo poſited as above, and Liquor poured in until it touches the Top-Diameter at the Edge B, the Surface of that Liquor may quor may be repreſented by the Line B*g*; by which it appears, that the Part of the Tun A B*g*, will be empty, and ſo the Tun will want ſo much of its whole Content; but to know how much that is ſhould be our next Buſineſs.

In order to do this, we ſhould know how many Inches and Parts the Surface of the Liquor at *g* wants of the Top of the Tun at A; which you may eaſily find (if the Tun be filled up to B,) by meaſuring from A, to the Liquor at *g*. But if there is no Liquor in the Tun, you may proceed to know where the Point *g* would fall (nevertheleſs) by the Directions following.

Take a Quadrant, and direct the Sights by the Top of the Tun at B, to the oppoſite Side AC, ſtill elevating or depreſſing the ſame, until the Thread *o* N, falls on the Beginning of the firſt Degree on the Limb; which it will do when you ſee the white Speck at *g*, made with Chalk, &c. or you may faſten a Line at B, and extend it to the oppoſite Side AC, holding it very tight; then bring it nearer or farther from the Top at A, until applying the Horizontal *Radius* of your Quadrant to the ſaid Line, the Thread may fall on the Beginning of the firſt Degree, which it will not do till brought to the Point *g*, as before.

Having thus found the Point *g*, you may with your Rule take its true Diſtance from the Point A, which we'll ſuppoſe to be 18 Inches. So have you the top Diameter AB, and Slanting-ſide A*g*, of the Fruſtum of a Cone AB*go*, to find the bottom Diameter *g o*, and perpendicular Depth A*t*; which you may truly do by Calculation, according to the Directions following.

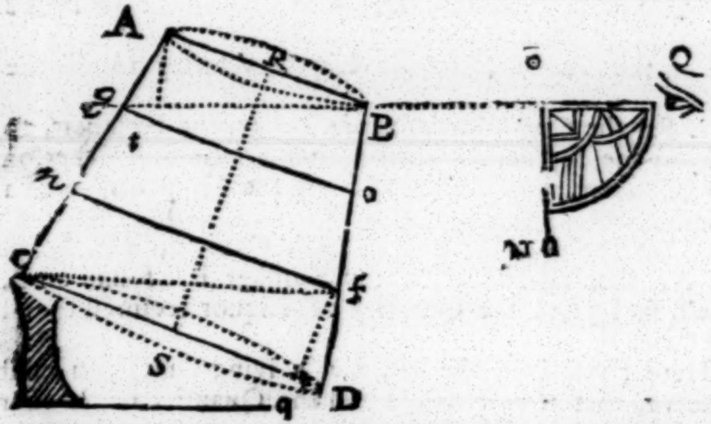
The Reaſon of which depends upon *Prop.* 4th of *Euclid* 6th (as above cited) where 'tis demonſtrated, that the Sides of equiangular Triangles are proportional.

## *To find the Drip or Fall of a Tun.*

Suppoſe a conical Tun (as ABCD) to be ſo poſited, as to have its Axis RS inclining towards the Horizon, ſo that when the Liqour juſt touches the Angle at C, (made by the bottom Diameter CD, and Side of the Cone AC,) it may wet 20,14 Inches of the oppoſite Side BD, *viz.* D*f*, or when filled ſo as to touch B, may want 18 Inches (on the oppoſite Side) of the Top as A*g*; let the Dimenſions of the Tun be firſt,

| | | |
|---|---|---|
| The Top Diameter AB | 90, | Inches. |
| Bottom Diameter CD | 106, | |
| Depth RS | 60, | |
| Slanting-ſide AC or BD | 60,53 | |

| | |
|---|---|
| According to theſe Dimenſions, the Content of the Tun in Beer Gallons and Parts is | 1608,69 Gallons. |



When this Tun is (for Conveniency of cleanſing) ſo poſited as above, and Liquor poured in until it touches the Top-Diameter at the Edge B, the Surface of that Liquor may [illegible] may be repreſented by the Line B*g*; by which it appears, that the Part of the Tun A B*g*, will be empty, and ſo the Tun will want ſo much of its whole Content; but to know how much that is ſhould be our next Buſineſs.

In order to do this, we ſhould know how many Inches and Parts the Surface of the Liquor at *g* wants of the Top of the Tun at A; which you may eaſily find (if the Tun be filled up to B,) by meaſuring from A, to the Liquor at *g*. But if there is no Liquor in the Tun, you may proceed to know where the Point *g* would fall (nevertheleſs) by the Directions following.

Take a Quadrant, and direct the Sights by the Top of the Tun at B, to the oppoſite Side AC, ſtill elevating or depreſſing the ſame, until the Thread *o* N, falls on the Beginning of the firſt Degree on the Limb; which it will do when you ſee the white Speck at *g*, made with Chalk, *&c.* or you may faſten a Line at B, and extend it to the oppoſite Side AC, holding it very tight; then bring it nearer or farther from the Top at A, until applying the Horizontal *Radius* of your Quadrant to the ſaid Line, the Thread may fall on the Beginning of the firſt Degree, which it will not do till brought to the Point *g*, as before.

Having thus found the Point *g*, you may with your Rule take its true Diſtance from the Point A, which we'll ſuppoſe to be 18 Inches. So have you the top Diameter AB, and Slanting-ſide A*g*, of the Fruſtum of a Cone AB*go*, to find the bottom Diameter *g o*, and perpendicular Depth A*t*; which you may truly do by Calculation, according to the Directions following.

The Reaſon of which depends upon *Prop.* 4th of *Euclid* 6th (as above cited) where 'tis demonſtrated, that the Sides of equiangular Triangles are proportional.

## *To find the Diameter* go

### *RULE.*

| | |
|---|---|
| From the bottom Diameter CD, *viz.* | 106 Inches. |
| Subſtract the top Diameter, *viz.* AB= | 90 |
| And there will remain | 16 |
| One Half of which is | 8 |

Then ſay, as the Slanting-ſide $AC=60,53$, is to Half the Difference of the Diameters AB and $CD=8$; ſo is the Slanting-ſide $Ag=18$, to Half the Difference of the Diameters AB, and $go=2,37$, which doubled, and added to the Diameter $AB=90$, will be 94,74, the Diameter *go* required.

## *To find the Depth* A *t*, *of the Fruſtum* ABgo.

### *RULE* I.

Say, as the Slanting-ſide $AC=60,53$, is to the Depth of the whole Tun $RS=60$; ſo is the Slanting-ſide $Ag=8$, to its proper Part of the Depth, *viz.* $At=17,84$ required.

Having thus found the Depth A*t*, and Diameter *go*, of the Fruſtum AB *go*, you may find what the Part AB *g* A will contain by the following

### *RULE* II.

To the Square of the top Diameter $AB=90$, add one Half of the Rectangle, or Product of the top and bottom Diameters; multiply the Sum by the Depth, (A *t*) and divide the Product by 1077 for Beer Gallons, and 882 for Wine.

*EX-*

## EXAMPLE.

The Square of the top Diam. *viz,* AB is 8100
Half the Product of *g o*=94,74 into AB=90 is 4263,30

Their Sum is 12363,30
Which multiply by the Depth A*t*= 17,84

4945320
9890640
8654310
1236330

1077) 220561,272 (204,792 Gallons.
005163348
515948
08922
000

The Content of the whole Tun ABCD in Ale Gallons is 1208,69
The Content of the Part AB*g* A 204,792

The Tun when fill'd to the Edge B, contains 1003,898

If you'd know how many Gallons is contained in the Part B*g o* B, being the lower Part of the Fruſtum AB *g o*, you may he ſatisfied by the following

## RULE III.

To the Square of the bottom Diameter, (*g o.*) add Half the Rectangle or Product of the bottom Diameter into the top Diameter; (*i e.* $\frac{1}{2}$ *g o*+AB) then multiply the Sum by the Depth, and divide the Product by 1077 for Beer Gallons, and 882 for Wine.

### EXAMPLE.

The Sq. of the bot. or lower Diam. 94=94,74 is 8975,6676
Half the Rect. of the Dia. AB=90, in 90=94,74 is 4263,30

Their Sum is 13238,9676
Which you muſt multiply by the Depth A*t*=17,84, and the Product will be } 236183,181984

Divide it by 1077, and the Quotient in Ale Gallons is 219,29729; and ſo much will the Part B*g**o*B contain, Q, E, F.

## *To find how much Liquor muſt be poured into the Tun (as it now ſtands) to cover the Bottom* CD, *take the Rules following.*

To do this, you muſt firſt (by the preceding Directions) find A*g*, which is equal to B*o*, and the Diameter *go*; which done, you may proceed to find D*f*: Thus,

Say, as *go*=94,74, is to CD=106, ſo is B*o*=A*g*=18, to D*f*=20,14; and ſo much of the Side BD will be wet, when the Liquor comes to touch at C.

## *To find the Diameter* f w.

Firſt add WC=*f*D=20,14, to A*g*=B*o*=18, and their Sum is 38,14; which take from AC=BD=60,53, and there will remain *g w*=*f*O=22,39=*f w*. Add to A*g*=B*o*=18, and the Sum will be A*w*=B*f*=40,39. Then ſay,

As AC=BD=60,53, is to Half the Difference of the Diameters, (*viz* AB=*go*, and CD=106,) *i. e.* 8, ſo is A*w*=B*f*=40,39, to Half the Difference of the Diameters AB and *wf*, *i. e.* 5,34; which doubled, and added to AB=*go*, gives 100,68 for the Diameter *ſw* required.

## *To find the Depth* fq, *of the Fruſtum* wf CD, *ſay*,

As BD=AC=60,53, is to the Depth of the Tun, *viz* RS=60, ſo is *ſ* D=20,14, to *ſ*,*q*=19 96. Now to find the Area of the Part DC *ſ*, you muſt conſult the ſecond Rule before-going, which was given to find the Part B *go* B: Thus,

| | |
|---|---|
| The Squ. of the bot. Diameter, *viz*. DC=106 is | 11236 |
| Half the Pro. of the Di. CD=106 in *wſ*=100,68 is | 5336,04 |
| Their Sum is | 16572,04 |
| Which multiply'd by the Depth *ſq*= | 19,96 |
| Produceth | 330777,9184 |
| Which Product divide by 1077, and the Quotient will be the Content of DC*ſ* in Ale Gallons. } *viz*. | 307,1289 |

But if you would find the Content of the other Parts, *viz*. C*ſw*, you may conſult the firſt Rule which was given to find the Part AB*g*A, as below.

## *To find the Content of the upper Part of this Fruſtum*, viz. C*ſw*.

The Square of the top Diam. $wf$ = 100,68 is 10136,4624
Half the Product of the Diam. $wt$ = 100,68 }
into CD = 106 is } 5336,04

Their Sum is 15472,5024
Which multiply by the Depth $fq$ = 19,96 19,96

928350144
1392525216
1392525216
154725024

And the Product is 308831,147904
Which divide by 1077, and the Quotient will be the Content of the Part $Cfw$ in Ale Gallons. } *i. e.* 286,7516

I have inſiſted upon this Matter, more than I would have done, had I not been importuned ſo to do, by ſome Gentlemen, whoſe Curioſity was ſuch, that they would find all, or as much as poſſible could be found, by Calculation; and becauſe 'tis what I never did ſee in any other Author, I thence concluded, that I might oblige others in ſo doing, as well as my Friends. And though ſome may think what is here done by Calculation, might have been found by extending Lines, and meaſuring more Diſtances than from A to *g*, with more Eaſe, *&c.* yet if the Line *Ag* be truly taken, and alſo the top and bottom Diameters with the Depth of the Tun; what elſe is to be found, may be had this Way, with more Eaſe and Exactneſs than any other; nay, and looks more like Art; for what would a Man (who is ignorant in Mathematical Learning) think, if he ſhould ſee an Officer pour Liquor by a known Meaſure into the Tun, till he had covered the Bottom? He would know it was but a Shift. And to uſe a Maſon's Level, Compaſſes, *&c.* is altogether as mechanical.

*How*

## *How the Diviſors, hitherto made uſe of, may be found, by knowing what Proportion their reſpective Figure has to its circumſcribing Cylinder or Parallelopipedon.*

A Cylinder is $\frac{177,5}{226}$ of the Parallelopipedon that circumſcribes it, and 282 and 231 are the Number of Inches contained in an Ale or Wine Gallon, according to the Statute, and the proper Diviſors for a Parallelopipedon; I demand the Diviſors for a Cylinder.

Say, as 177,5 the Numerator, is to 226 the Denominator; ſo is the Diviſor for a Parallelopipedon, to the Ale Diviſor for a Cylinder.

```
As 177,5 : 226 : : 6 282
                       226
                    ------
                      1692
                      564
                     564
                   -------
         177,5) 63732,0000   (359,053 Ale Divi-
                 104875055     ſor for a Cylind.
                  1609522
                    00969
                      00
```

For

### *For the Wine Diviſor, ſay,*

As 177,5 : 226 : : 231 the Wine Diviſor for a Parallelopipedon.
226

1386
462
462

177,5) 52206,0000 (294,118 the Wine Diviſor for a Cylinder.
167010550
0731275
02345
010

### *For a Malt Diviſor.*

2150,419 being the Diviſor for a Parallelopipedon.

As 177,5 : 266 : : 2150,419 : 2737,99, *&c.* the Diviſor for a Circle Cylinder, *&c.* But in the practick Part of this Art, 'tis agreed, that 2737,47 muſt be uſed, as is before-noted.

A Cone that is circumſcribed by a Cylinder of the ſame Diameter and Heighth is one third thereof, ***Vid. Hayes's Mathem. Philoſ,*** Page 161.

## PROB. II.

*A Cone being $\frac{1}{3}$ of a Cylinder, I demand the Beer, Wine, and Malt Diviſors for a Cone; thoſe of the Cylinder being for Beer* 359,053, *for Wine* 294,118, *and* 2737,47.

### *For the Wine Diviſor.*

As 1 : 3 : : 294,118
3

882,354 Wine Diviſor for a Cone.

*For*

### For the Beer Diviſor.

As 1 : 3 : : 359,053 :
3

1077,159 The Beer Diviſor for a Cone.

### For the Malt Diviſor.

As 1 : 3 : : 2737,99 : 8213,07 But you may uſe
3 8212,41 (if an Officer.)

8213,97

I ſhall inſiſt no farther in theſe Examples, becauſe what is ſaid of the Cone is ſufficient to inform the meaneſt Capacity how to proceed in the Globe, Spheroid, Square, Pyramid, parabolick Conoid, hyperbolick Conoid, and parabolick Spindle, when he knows what Part of their circumſcribing Cylinder they are; and when that is known, the Proportion will be as the Numerator is to the Denominator, ſo is the Beer, Wine, and Malt Diviſors of a Cylinder, to the like Diviſors for that Figure.

A Cylinder is $\frac{11}{14}$ of the Parallelopipedon that circumſcribes it, or (nearer) $\frac{177,5}{226,}$ of it as above.

A Cone is $\frac{1}{3}$ of its circumſcribing Cylinder.

A Globe is $\frac{2}{3}$ of the Cylinder that circumſcribes it, having equal Diameters and Heighth.

A Spheroid is $\frac{2}{3}$ of the Cylinder, whoſe Conjugate is the Diameter, and tranſverſe Diameter the Depth of the Cylinder.

As a Cone is $\frac{1}{3}$ of a Cylinder, ſo a ſquare Pyramid is $\frac{1}{3}$ of that Parallelopipedon that circumſcribes it, having the ſame Baſe and Altitude; and in general, let the Baſe of a Pyramid be what you pleaſe, *i.e.* ſquare, circular, triangular, polygonal, *&c.* they will be but $\frac{1}{3}$ of their circumſcribing Priſms, that have the ſame Baſe and Heighth.

A parabolick Conoid is $\frac{1}{2}$ of the Cylinder that circumſcribes it, ſuppoſing it to have the ſame Baſe and Altitude with that of the Cylinder.

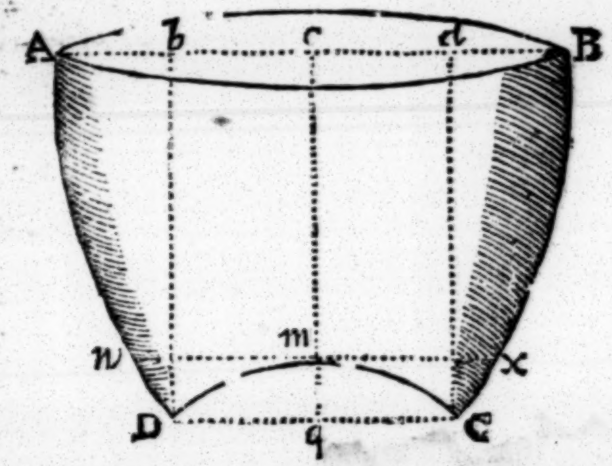
A hyperbolick Conoid is $\frac{5}{12}$ of the Cylinder that circumſcribes it, having the ſame Baſe and Altitude as the Cylinder.

A parabolick Spindle is $\frac{8}{15}$ of that Cylinder, whoſe Axis is the ſame with the parabolick Spindle, and Diameters equal.

He that would know more of the Nature of theſe Figures, may have Recourſe to that ingenious Piece of Work done by Mr. *Hayes*, Entituled, ***Mathematical Philoſophy***, where from Page 161 to Page 177, they may be fully ſatisfied.

## *To gauge a Copper, and make Allowance for the Crown.*

Let ABCD repreſent a Copper to be gauged, the Dimenſions may be taken, and the Content found, as below.



1. Firſt faſten a ſmall Chord or Thread at A, and extend the other End diametrically to the Point B, where faſten it alſo; then ſet one End of your Inſtrument on D, and move the upper End thereof towards A or $c$, till you find the neareſt Diſtance of the Point D, to the Thread AB, which will be when moved to $b$; let $b$D be 40 Inches, the Depth of the Furnace or Copper.

2. In like manner ſet one End of your Inſtrument upon the top of the Crown at $m$, and take the neareſt Diſtance of that Point to the Line AB, which let be $c\,m = 30$ Inches; which take from $Db = Cd = 40$, and there will remain the Depth or Heighth $m\,q = 10$ Inches.

## *To find the Diameter of the Crown* DC, *meaſure* AB, *and call it* 126 *Inches.*

Then take a Thread and Plummet, and ſetting one End thereof upon the Line AB, draw it along that Line from A towards B, until the Plummet hangs juſt over the Point D, which it will do when at $b$; then meaſure A$b$, and ſuppoſe it to be 12,3 Inches; then find the Diſtance B$d$, and let that be 12,3 Inches alſo: Add theſe together, and their Sum is 24,6, which take from the Diameter at the Top, *viz.* AB = 126, and there will remain the Diameter of the Crown DC = 101,4 Inches.

Now to find the Content of the Copper from the Crown upwards, (that is, the Part AB $wx$,) the Depth ($mc$) being 30 Inches, you may find a Diameter in the Middle of every 5 Inches of the Depth $mc$; which having found, you may ſet them againſt their reſpective Depths, as in the ſecond Column of the following Table: Then find (by the Ale Gauge-point upon the Sliding-Rule) the Area's of thoſe ſeveral Diameters, and ſet them againſt their proper Diameters in the third Column: Having ſo done, the fourth Column is made by multiplying each Area by 5, ſo have you the Content of the Part AB$wx$, at

every 5 Inches of the Depth *m c*, which added together makes
B. F. Gall.
32 : 00 : 3,3.

| Parts of the Depth | Diameters. | The Areas of thoſe Diam. | Content of every Inches reduc'd to. B. F. Gall. |
|---|---|---|---|
| 5 | 124,8 | 43,38 | 6 : 01 : 4,4 |
| 5 | 119,9 | 40,04 | 5 : 03 : 4,7 |
| 5 | 115,7 | 37,3 | 5 : 01 : 8,0 |
| 5 | 111,3 | 34,5 | 5 : 00 : 2,5 |
| 5 | 108,8 | 32,97 | 4 : 03 : 3,4 |
| 5 | 103,9 | 20,06 | 4 : 01 : 5,8 |
| 30 | Sum is | | 32 : 00 : 3,3 |
| To cover the Crown | | | 4 : 03 : 5,8 |
| Content of the Copper | | | 36 : 03 : 9,1 |

## *For the Content of the Part w x* DC.

The Diameter *w x* is 110,6, and DC = 101,4, and the Depth *m g* is 10.

The Area of the top Diam. *viz.* *w x* = 110,6 is 34,0734
The Area of the bottom Diam. *viz.* DC = 101,4 28,64
A Geometrical Mean by the Sliding is 31,24
Which multiply'd by the Deprh *m q* = 10,
The Pro. will be the Content required, *viz.* 312,40 Gall.

## *For the Content of the Part* D *m* C *q* D.

If we ſuppoſe this Part to be the Fruſtum of a Globe; its Diameter DC being 101,4 Inches, and Altitude *m q* = 10.

The Content (by the Rule herein before given for this Purpoſe) will be found to be 145,0565 Gallons.

Now if we ſubſtract the Content of the Crown (D*m* C*q* D) laſt found, *viz.* 145,0565 from rhe Content of the Part *w x* DC, *viz.* 312,4, there will remain 167, 3435 Gallons, and ſo much Liquor the Crown will take to cover it; which is 4 Bar. 3 Fir. 5,8 Gall. Add this to the
B. F. Gall.
Part AB *w x*, *viz.* 32 : 00 : 3,3, and the Sum will be the Content of the Furnace or Copper AD *m* CB, *viz.*
B. F. Gall.
36 : 03 : 9,1, which was required, and the ſame may be done in any other the like Caſe. And note, that the more curved the Sides of any Figure is, the more mean Circles you ought to take, *i.e.* in the Middle of every 8, 6, or 4 Inches, *&c.*

## *Having the Diameters and Depth of any conical Tun, to find the Diameter at any Depth at one Operation.*

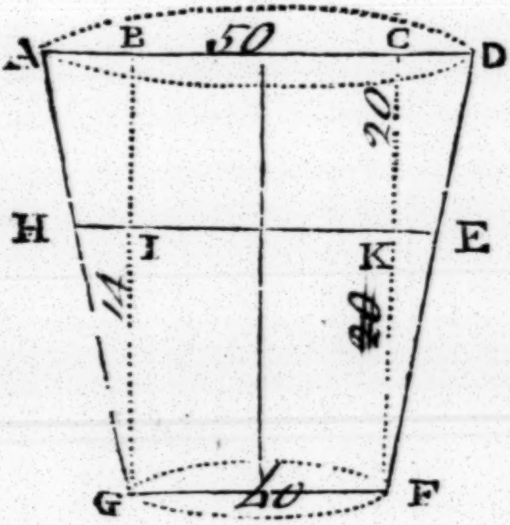
### *RULE.*

Say, as the Depth of the Tun is to half the Difference of the bottom and top Diameters, ſo is any Depth to half the Difference between the bottom Diameter, and that at the Depth propoſed; which doubled and added to the bottom Diameter, the Sum will be the Diameter required.

### *EXAMPLE* I.

Let ABCDEFGH repreſent a Tun in the Form of the Fruſtum of a Cone ſtanding on its leſſer Baſe; let the Diameter AD be 50 Inches, the Diameter GF 40 Inches, and the Depth BG or CF 20, and the given Depth KF or IG = 14 Inches to find the Diameter HE.

The Triangles CFD, and KFE being Similar, the Proportion will be as before, *i. e.* as the Depth of the Tun CF = 20, is to DC half the Difference of AD, and FG = 5; so is KF = 14, to KE half the Difference of GF and HE, *viz.* 3,5, which doubled is 7, and this added to the bottom Diameter GF, gives 47 for the Diameter HE required; and the like may be done for the Diameter at any other Depth, when the Tun ſtands upon its leſſer Baſe.

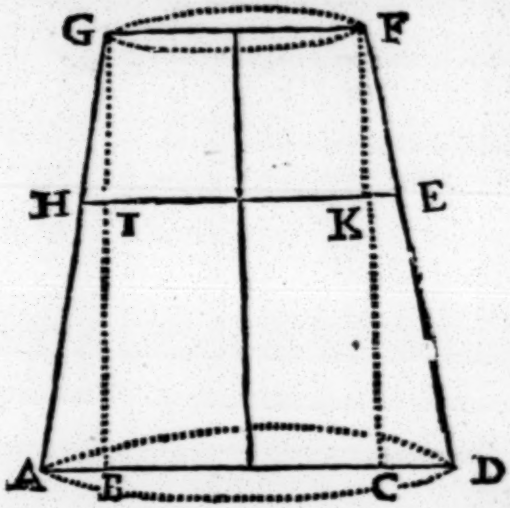


### *EXAMPLE* II.

Let a Tun having the ſame Dimenſions, be given only in another Poſition, *i. e.* ſtanding upon its greater Baſe; and it be required to find the Diameter at 14 Inches from the Bottom, as above.

I demand the ſaid Diameter.

### *RULE.*



### *RULE.*

Say, as the Depth FC = 20 Inches, is to Half the Difference of the Diameters GF and AD, *i e.* CD = 5, ſo is FK = 6, (*i.e.* the Diſtance from the Top to the Diameter HE required) to KE = 1,5, which doubled is 3, and this added to 40 = GF, is 43 for the Diameter HE ſought; and the ſame Way of working muſt be obſerved to find the Diameter at any other Depth, when the Tun is in this Poſition. Where note, that in either of theſe Figures we may ſuppoſe the Line HE to be the Diameter of the Liquor's Surface, which, with the Depth 14 and bottom Diameter, the true Quantity of Liquor may be found which is now in the Tun, or which it will hold at 14 Inches Deep.

## *To find how many Gallon, a conical Tun will hold upon veery Inch of the Depth.*

Let the Diameter at the Top be 80 18 Inches.
The Diameter at the Bottom 98
Depth 40

The Content in Ale Gallons is 885,5, how many Ale Gallons will this Tun hold upon every Inch of its Depth?

### *R U L E.*

Divide the Difference of the Diameters by the Depth, and the Quotient is a common Addend for one Inch from Top to Bottom; by which you may find the Diameter in the Middle of every 10 Inches of the Depth, as below.

Bottom Diameter 98
Top Diameter 80
Their Difference 18 for a Dividend, which
divide by 40) 18,00 (,45 the Addend for one Inch.
20(0
0

Now 5, 15, 25 and 35, are the Inches which fall in the Middle of every 10 Inches of the Depth, which Depths, if you multiply by the common Addend, will ſhew what muſt be added to the top Diameter, to find the Diameters at thoſe Depths. Thus,

| | | | |
|---|---|---|---|
| 5, | 15, | 25, | 35, |
| ,45 | ,45 | ,45 | ,45 |
| 2,25 | 6,75 | 11,25 | 15,75 |

80, Top Diameter
2,25

Gall. Bar. F. Gall.
82,25 Diam. at 5 Inch. Deep, its Area 18,84 = 00 : 2 : 1,84
80, Top

80, Top Diameter
6,75

Gall. Bar. F. Gall.
86,75 Diam. at 15 Inch. Deep, its Area 20,96 = 00 : 2 : 3,96

80, Top Diameter
11,25

91,25 Diam. at 25 Inch. Deep, its Area 23,19 = 00 : 2 : 6,19

80, Top Diameter
15.75

95,75 Diam. at 35 Inch. Deep, its Area 25,53 = 00 : 3 : 0,03

The Sum of the Areas are 88,52 which multiplied by 10, is 885,2 the Tun's Content.

Bar. Fir. Gall.
Now 18,84 Gallons, or 00 : 02 : 1,84 being an Area common to any of the firſt ten Inches; you are to ſubſtract either of them from the Tun's Content in the like Name continually, and the Remainder will be the Area at each of the firſt ten Inches; and then for the next
Bar. Fir. Gall.
ten Inches you muſt take 20,96 or 00 : 02 : 3,96 out of the laſt Remainder, *&c.* as in the following Table.

---

Bar. Fir. Gall. Pts.
The whole Content is 885,2 Gallons, or 26 : 00 : 1,20
02 : 1,84
25 : 01 : 7,86
02 : 1,84
24 : 03 : 6,02
02 : 1,84
24 : 01 : 4,18

| Bar. | Fir. | Gall. Pts. |
|---|---|---|
| 24 | 01 | 4,18 |
|  | 02 | 1,84 |
| 23 | 03 | 2,34 |
|  | 02 | 1,84 |
| 23 | 01 | 0,50 |
|  | 02 | 1,84 |
| 22 | 02 | 7,16 |
|  | 02 | 1,84 |
| 22 | 00 | 5,32 |
|  | 02 | 1,84 |
| 21 | 02 | 3,48 |
|  | 02 | 1,84 |
| 21 | 00 | 1,64 |
|  | 02 | 1,84 |
| 20 | 01 | 8,30 |
|  | 02 | 3,96 |
| 19 | 03 | 4.34 |
|  | 02 | 3.96 |
| 19 | 01 | 0,38 |
|  | 02 | 3,96 |
| 18 | 2 | 4,92 |
|  | 2 | 3,96 |
| 18 | 0 | 0,96 |
|  | 2 | 3,96 |
| 17 | 1 | 5,50 |
|  | 2 | 3,96 |
| 16 | 3 | 1,54 |
|  | 2 | 3,96 |
| 16 | 0 | 6,08 |
|  | 2 | 3,96 |
| 15 | 2 | 2,12 |

Note,

Bar. Fir. Gall. Pts.

15 : 2 : 2,12
2 : 3,96

14 : 3 : 6,66
2 : 3,96

14 : 1 : 2,70
2 : 6,19

Note,

13 : 2 : 5,01
2 : 6,19

12 : 3 : 7,32
2 : 6,19

12 : 1 : 1,13
2 : 6,19

11 : 2 : 3,44
2 : 6,19

10 : 3 : 5,75
2 : 6,19

10 : 0 : 8,06
2 : 6,19

9 : 2 : 1,87
2 . 6,19

8 : 3 : 4,18
2 : 6,19

8 : 0 : 6,49
2 : 6,19

Note,

7 : 2 : 0,30
3 : 0,03

6 : 3 : 0,27

| Bar. | Fir. | Gall.Pts. |
|---|---|---|
| 6 : | 3 : | 0,27 |
| | 3 : | 0,03 |
| 6 : | 0 : | 0,24 |
| | 3 : | 0,03 |
| 5 : | 1 : | 0,21 |
| | 3 : | 0 03 |
| 4 : | 2 : | 0,18 |
| | 3 : | 0,03 |
| 3 : | 3 : | 0,15 |
| | 3 : | 0,03 |
| 3 : | 0 : | 0,12 |
| | 3 : | 0,03 |
| 2 : | 1 : | 0,09 |
| | 3 : | 0,03 |
| 1 : | 2 . | 0.06 |
| | 3 : | 0.03 |
| 0 : | 3 : | 0,03 |
| | 3 : | 0,03 |
| 0 : | 0 : | 0.00 |

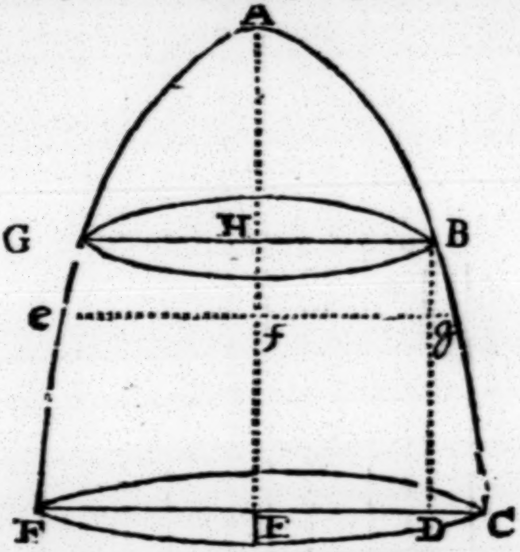
Bar. Fir. Gall. Pts.

The Table following is collected from the preceding Work, whoſe Uſe is to find how much Liquor is in ſuch a Tun, by having the Diſtance of the Liquor's Surface from the Top: Thus, let it be 20 Inches, look 20 in the firſt Column, and againſt it you'll find 14 : 01 : 2,70, *i. e.* 14 Barrels, 1 Firkin, 2 Gallons, and ſeven tenth Parts, and ſo for any other Fruſtum of a Pyramid.

Inches.

| Inches | Barrels. | Firkins. | Gall. Pts. |
|---|---|---|---|
| 0 | 26 | 00 | 1,20 |
| 1 | 25 | 01 | 7,86 |
| 2 | 24 | 03 | 6,02 |
| 3 | 24 | 01 | 4,18 |
| 4 | 23 | 03 | 2,34 |
| 5 | 23 | 01 | 0,50 |
| 6 | 22 | 02 | 7,16 |
| 7 | 22 | 00 | 5,32 |
| 8 | 21 | 02 | 3,48 |
| 9 | 21 | 00 | 1,64 |
| 10 | 20 | 01 | 8,30 |
| 11 | 19 | 03 | 4,34 |
| 12 | 19 | 01 | 0,38 |
| 13 | 18 | 02 | 4,92 |
| 14 | 18 | 00 | 0,96 |
| 15 | 17 | 01 | 5,50 |
| 16 | 16 | 03 | 1,54 |
| 17 | 26 | 00 | 6,08 |
| 18 | 15 | 02 | 2,12 |
| 19 | 14 | 03 | 6,66 |
| 20 | 14 | 01 | 2,70 |
| 21 | 13 | 02 | 5,01 |
| 22 | 12 | 03 | 7,32 |
| 23 | 12 | 01 | 1,13 |
| 24 | 11 | 02 | 3,44 |
| 25 | 10 | 03 | 5,75 |
| 26 | 10 | 00 | 8,06 |
| 27 | 9 | 02 | 1,87 |
| 28 | 8 | 03 | 4,18 |
| 29 | 8 | 00 | 6,49 |
| 30 | 7 | 02 | 0,30 |
| 31 | 6 | 03 | 0,27 |
| 32 | 6 | 00 | 0,24 |
| 33 | 5 | 01 | 0,21 |
| 34 | 4 | 02 | 0,18 |
| 35 | 3 | 03 | 0,15 |
| 36 | 3 | 00 | 0,12 |
| 37 | 2 | 01 | 0,09 |
| 38 | 1 | 02 | 0,06 |
| 39 | 0 | 03 | 0,03 |
| 40 | 0 | 00 | 0,00 |

## *To find the Diameter at any Depth of a Tun, in the Form of the Fruſtum of a parabolick Conoid.*



### DEFINITIONS.

If we conceive the curve Line FGABC to repreſent a Parabola; then,

HB is called a Semiordinate, and AH its Abſciſſa; and ſo EC is a Semiordinate, and AE its Abſciſſa.

The Latus-Rectum of a Parabola (as of this) is ſuch a Line, as that being multiplied by the Abſciſſa AH, the Rectangle or Product will be equal to the Square of HB.

If you multiply it by the Abſciſſa AE, then the Rectangle will be equal to the Square of EC, and will always be ſo, as in *Apoll. Prop.* 11. *Lib.* 1.

Now to find the Latus Rectum of the Parabola by having only the top Diameter $GB=40$, the bottom Diameter $FC=50$, and the Depth HE or $BD=20$, muſt be performed by that Property of the Parabola, commonly called *Baker's Property*, whereon his *Clavis Geometrica Catholica* depends, and is this.

In the Parabola, the Latus Rectum is to the Sum of any two Semiordinates, as their Difference is to the Difference of their Abſciſſa; that is in this preſent Caſe, as L (the Latus Rectum) is to $HB + EC = FD$, ſo is DC to BD: And by changing the Conſequents to Antecedents, the Proportion will ſtand thus:

As $BD : DC :: FD : L$, that is, in Words, as the Depth of the Tun (BD) 20 is to half the Difference of the Diameters $DC = 5$, ſo is half the Sum of the Diameters (GB and FC) 45 to 11,25, and ſo much is the Latus Rectum of the Parabola belonging to the Figure.

Having thus found the Parameter or Latus Rectum, we are next to find the Length of the Line AH, the proper Abſciſſa to the Semiordinate HB; to do which, you are to know, that the Square of HB is equal to the Rectangle of the Latus Rectum and Abſciſſa AH. Therefore divide the Square of $HB = 400$, by the Latus Rectum 11,25, and the Quotient will be 35.55555 of the Abſciſſa AH; to which add $6 = Hf$, and the Sum is $Af = 41{,}55555$, or the proper Abſciſſa of the Semiordinate $fg$, which is now only wanting. But knowing its Square is equal to the Rectangle of the Latus Rectum and Abſciſſa $Af$, multiply 41,55555 by 11,25, and the Product is———467,4999375, which is the Square of the Semiordinate $fg$; the Square Root of which is 21, 6217, which doubled is 43,2434, the Diameter $eg$ required; and ſo of any other. This will not appear difficult to any who underſtand ſo much of the Conick Sections as is requiſite for the Conſtruction of Algebraick Equations.

Many Things of this Nature I might inſiſt upon, as in ſhowing how to find the Diameters at every Inch of this Depth, of this, or any other regular Tun: But what has been ſaid already, may be ſufficient to excite the induſtrious Reader to peruſe others who have taken more Room in Writing to that Purpoſe, than I have Time to do at preſent. But if what is here done proves any way ſerviceable, I have my Deſire; and by knowing it to be approved of, ſhall be encouraged thereby to publiſh ſome things in the Art of Gauging, as may oblige Poſterity.

A

## A TABLE of *Circles*, *Areas*, in Malt Buſhels and decimil[redacted] Parts, calculated to every tenth Part of an Inch of the Diameter from 12 to 100 Inches.

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 12 | 0,0526 | 0,0534 | 0,0543 | 0,0552 | 0,0561 |
| 13 | 0,0617 | 0 0626 | 0,0636 | 0,0646 | 0.065 |
| 14 | 0,0716 | 0,0726 | 0,0736 | 0,0747 | 0,0757 |
| 15 | 0.0822 | 0.0833 | 0,0844 | 0,0855 | 0,0866 |
| 16 | 0,0935 | 0,0946 | 0,0957 | 0,0970 | 0,0982 |
| 17 | 0,1055 | 0,1068 | 0,1080 | 0,1093 | 0,1106 |
| 18 | 0,1183 | 0 1196 | 0,1210 | 0,1223 | 0,1236 |
| 19 | 0 1318 | 0,1332 | 0,1346 | 0,1360 | 0,1374 |
| 20 | 0,1461 | 0,1475 | 0,1490 | 0,1505 | 0,1520 |
| 21 | 0.1611 | 0,1626 | 0,1642 | 0,1657 | 0,1670 |
| 22 | 0,1768 | 0,1784 | 0,1800 | 0,1816 | 0,1833 |
| 23 | 0.1932 | 0,1949 | 0.1966 | 0,1983 | 0.2000 |
| 24 | 0,2104 | 0,2121 | 0,2139 | 0,2157 | 0,2174 |
| 25 | 0.2283 | 0.2301 | 0,2319 | 0,2338 | 0.2356 |
| 26 | 0 2469 | 0,2488 | 0,2507 | 0,2526 | 0,2546 |
| 27 | 0.2663 | 0,2682 | 0,2702 | 0,2725 | 0,2758 |
| 28 | 02864 | 0,2884 | 0,2904 | 0,2925 | 0,2946 |
| 29 | 0 3072 | 0,3093 | 0,3114 | 0 3136 | 0,3151 |
| 30 | 0,3287 | 0,3309 | 0 3331 | 0,3353 | 0 3376 |
| 31 | 0.3512 | 0.3516 | 0.3556 | 0.3578 | 0.3601 |
| 32 | 0,3740 | 0 3764 | 0,3787 | 0.3810 | 0,3834 |
| 33 | 0,3978 | 0,4002 | 0,4026 | 0,4050 | 0 4075 |
| 34 | 0 4222 | 0 4247 | 0.4272 | 0,4297 | 0,4322 |
| 35 | 0.4475 | 0.4500 | 0,4526 | 0,4552 | 0.4577 |
| 36 | 0 4734 | 0,4700 | 0,4787 | 0,4813 | 0,4840 |
| 37 | 0 5001 | 0.5028 | 0 5055 | 0,5082 | 0,5109 |
| 38 | 0 5275 | 0,5302 | 0,5330 | 0,5358 | 0,5386 |
| 39 | 0.5556 | 0.5684 | 0,5613 | 0,5642 | 0 5670 |

| 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| 0,0570 | 0,0579 | 0,0589 | 0,0598 | 0,0607 |
| 0,0657 | 0,0675 | 0,0685 | 0,0695 | 0,0705 |
| 0,0768 | 0,0778 | 0,0792 | 0,0800 | 0,0811 |
| 0,0877 | 0,0888 | 0,0900 | 0,0912 | 0,0923 |
| 0,0994 | 0,1006 | 0,1018 | 0,1031 | 0,1042 |
| 0,1118 | 0,1131 | 0,1144 | 0,1157 | 0,1170 |
| 0,1250 | 0,1263 | 0,1277 | 0,1291 | 0,1304 |
| 0,1389 | 0,1403 | 0,1417 | 0,1432 | 0,1446 |
| 0,1535 | 0,1550 | 0,1565 | 0,1580 | 0,1595 |
| 0,1688 | 0,1704 | 0,1720 | 0,1736 | 0,1752 |
| 0,1850 | 0,1866 | 0,1882 | 0,1899 | 0,1915 |
| 0,2017 | 0,2034 | 0,2052 | 0,2070 | 0,2086 |
| 0,2192 | 0,2210 | 0,2228 | 0,2246 | 0,2264 |
| 0,2375 | 0,2394 | 0,2412 | 0,2431 | 0,2450 |
| 0,2565 | 0,2584 | 0,2604 | 0,2623 | 0,2643 |
| 0,2762 | 0,2782 | 0,2803 | 0.2823 | 0,2843 |
| 0,2968 | 0,2988 | 0,3009 | 0,3029 | 0,3151 |
| 0,3178 | 0,3200 | 0,3215 | 0,3244 | 0,3265 |
| 0,3398 | 0,3420 | 0,3443 | 0,3465 | 0,3488 |
| 0,3624 | 0,3647 | 0,3670 | 0,3694 | 0,3717 |
| 0,3858 | 0,3882 | 0,3906 | 0,3930 | 0,3954 |
| 0,4099 | 0,4124 | 0,4148 | 0,4173 | 0,4198 |
| 0,4348 | 0,4373 | 0,4398 | 0,4424 | 0,4449 |
| 0,4603 | 0,4629 | 0,4655 | 0,4681 | 0,4708 |
| 0,4866 | 0,4893 | 0,4920 | 0,4947 | 0,4974 |
| 0,5137 | 0,5164 | 0,5192 | 0,5219 | 0,5247 |
| 0,5414 | 0,5442 | 0,5471 | 0,5499 | 0,5527 |
| 0,5699 | 0,5728 | 0,5757 | 0.5786 | 0,5815 |

## Areas of Circles in Malt Buſhels.

| D | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 40 | 0,5844 | 0.5874 | 0.5903 | 0,5932 | 0,5962 |
| 41 | 0,6140 | 0 6170 | 0,6200 | 0,6230 | 0,6261 |
| 42 | 0.6444 | 0.6474 | 0.6505 | 0,6536 | 0,6567 |
| 43 | 0,6754 | 0,6786 | 0,6817 | 0,6849 | 0,6880 |
| 44 | 0.7072 | 0.7104 | 0,7136 | 0,7169 | 0,7201 |
| 45 | 0.7397 | 0,7430 | 0,7463 | 0,7496 | 0,7529 |
| 46 | 0,7729 | 0,7763 | 0.7797 | 0,7831 | 0,7864 |
| 47 | 0,8069 | 0,8104 | 0,8138 | 0 8173 | 0,8207 |
| 48 | 0.8416 | 0,8451 | 0,8486 | 0,8522 | 0 8557 |
| 49 | 0,8770 | 0,8826 | 0,8842 | 0,8878 | 0,8914 |
| 50 | 0.9132 | 0,9169 | 0,9206 | 0,9242 | 0,9279 |
| 51 | 0,9501 | 0 9538 | 0,9576 | 0,9613 | 0.9650 |
| 52 | 0.9877 | 0,9915 | 0,9954 | 0,9992 | [illegible]0030 |
| 53 | 1.0261 | 1,0300 | 1,0338 | 1,0377 | 1,0416 |
| 54 | 1.0652 | 1.0691 | 1,0731 | 1,0770 | 1,0810 |
| 55 | 1.1050 | 1.1090 | 1.1130 | 1,1[illegible] | 1,1211 |
| 56 | 1,1455 | 1,1496 | 1,1537 | 1,1579 | 1,1620 |
| 57 | 1.1868 | 1,1910 | 1.1952 | 1,1593 | 1,2035 |
| 58 | 1,2288 | 1.2331 | 1.2373 | 1,2416 | 1,2458 |
| 59 | 1.2716 | 1.2759 | 1 2802 | 1,2845 | 1,2889 |
| 60 | 1.3150 | 1,3194 | 1,3238 | 1,3282 | 1,3326 |
| 61 | 1,3593 | 1,3637 | 1,3680 | 1,3726 | 1,3771 |
| 62 | 1.4042 | 1,4087 | 1,4133 | 1,4178 | 1,4224 |
| 63 | 1,4498 | 1,4544 | 1,4591 | 1,4637 | 1,4683 |
| 64 | 1.4962 | 1,5009 | 1,5056 | 1,5103 | 1,5150 |
| 65 | 1,5434 | 1,5581 | 1,5529 | 1,5576 | 1,5624 |
| 66 | 1.5912 | 1,5960 | 1,6009 | 1,6057 | 1,6100 |
| 67 | 1,6398 | 1,6447 | 1,6496 | 1,6545 | 1,6504 |
| 68 | 1,6891 | 1,6941 | 1,6991 | 1,7040 | 1,7090 |
| 69 | 1,7392 | 1,7442 | 1,7493 | 1,7543 | 1,7594 |
| 70 | 1,7899 | 1,7951 | 1,8002 | 1,8053 | 1,8104 |
| 71 | 1.8414 | 1.8466 | 1,8518 | 1,8570 | 1,8622 |

| 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| 0,5991 | 0.6021 | 0,6050 | 0,6081 | 0,6110 |
| 0,6291 | 0.6321 | 0,6352 | 0,6382 | 0,6444 |
| 0,6598 | 0,6629 | 0,6660 | 0,6691 | 0,6723 |
| 0,6912 | 0,6944 | 0,6976 | 0,6978 | 0,7040 |
| 0,7233 | 0,7266 | 0,7299 | 0,7331 | 0.7364 |
| 0,7572 | 0,7596 | 0762 9 | 0,7662 | 0,7696 |
| 0,7898 | 0,7932 | 0,7966 | 0,8001 | 0,8035 |
| 0,8242 | 0,8277 | 0,8311 | 0,8346 | 0,8381 |
| 0,8592 | 0,8625 | 0,8664 | 0,8699 | 0.8735 |
| 0,8950 | 0,8987 | 0,9023 | 0,9059 | 0,9096 |
| 0,9316 | 0,9353 | 0,9390 | 0,9427 | 0,9464 |
| 0,9688 | 0,9726 | 0,9764 | 0,9802 | 0,9839 |
| 1,0008 | 1,0110 | 1,0145 | 1,0184 | 1,0222 |
| 1,0455 | 1,0495 | 1,0534 | 1,0573 | 1,0612 |
| 1,0850 | 1,0890 | 1,0935 | 1,0970 | 1,1010 |
| 1,1252 | 1,1292 | 1,1333 | 1,1374 | 1,1415 |
| 1,1611 | 1,1702 | 1,1744 | 1,1785 | 1.1827 |
| 1,2077 | 1,2119 | 1,2162 | 1,2204 | 1,2246 |
| 1,2501 | 1,2544 | 1,2587 | 1,2630 | 1,2673 |
| 1,2934 | 1,2976 | 1,3019 | 1,3063 | 1.3107 |
| 1,3371 | 1,3415 | 1,3459 | 1,3503 | 1,3548 |
| 1,3816 | 1,3868 | 1,3906 | 1,3951 | 1.3997 |
| 1,4269 | 1,4315 | 1,4361 | 1,4406 | 1.4452 |
| 1,4729 | 1,4776 | 1.4822 | 1.4869 | 1.4916 |
| 1,5190 | 1,5244 | 1,5292 | 1,5339 | 1,5386 |
| 1,5672 | 1,5720 | 1,5768 | 1,5816 | 1,5864 |
| 1,6154 | 1,6203 | 1,6252 | 1,6300 | 1 6349 |
| 1,6644 | 1,6693 | 1.6742 | 1,6792 | 1,6841 |
| 1,7140 | 1,7191 | 1,7241 | 1,7291 | 1,7341 |
| 1,7645 | 1,7695 | 1,7746 | 1,7797 | 1,7848 |
| 1,8156 | 1,8208 | 1.8259 | 1,8311 | 1,8363 |
| 1,8675 | 1,8727 | 1,8779 | 1,8832 | 1,8884 |

| D. | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 72 | 1,8937 | 1,8989 | 1,9043 | 1,9095 | 1,9148 |
| 73 | 1,9466 | 1,9522 | 1,9573 | 1,9627 | 1,9680 |
| 74 | 2,0003 | 2,0058 | 2,0112 | 2,0166 | 2,0220 |
| 75 | 2,0548 | 2,0603 | 2,0658 | 2,0712 | 2,0768 |
| 76 | 2,1099 | 2,1155 | 2,1211 | 2,1266 | 2,1322 |
| 77 | 2,1658 | 2,1715 | 2,1771 | 2,1827 | 2,1884 |
| 78 | 2,2225 | 2,2282 | 2,2339 | 2,2396 | 2,2453 |
| 79 | 2,2798 | 2,2856 | 2,2914 | 2,2972 | 2,3029 |
| 80 | 2,3379 | 2,3437 | 2,3496 | 2,3555 | 2,3612 |
| 81 | 2,3967 | 2,4026 | 2,4085 | 2,4145 | 2,4204 |
| 82 | 2.4562 | 2,4622 | 2,4682 | 2,4742 | 2,4830 |
| 83 | 2,5165 | 2,5226 | 2,5287 | 2,5347 | 2,5408 |
| 84 | 2,5775 | 2,5837 | 2,5898 | 2,5960 | 2,6021 |
| 85 | 2,6393 | 2,6455 | 2,6517 | 2,6579 | 2,6642 |
| 86 | 2,7017 | 2,7080 | 2,7143 | 2,7206 | 2,7269 |
| 87 | 2,7649 | 2,7713 | 2,7776 | 2,7840 | 2,7904 |
| 88 | 2,8283 | 2,8353 | 2,8417 | 2,8489 | 2,8549 |
| 89 | 2,8935 | 2,9000 | 2,9065 | 2,9131 | 2,9196 |
| 90 | 2,9589 | 2,9655 | 2,9721 | 2,9787 | 2,9853 |
| 91 | 3,0250 | 3,0327 | 3,0383 | 3,0450 | 3,0517 |
| 92 | 3,0919 | 3,0986 | 3,1053 | 3,1121 | 3,1188 |
| 93 | 3,1595 | 3,1694 | 3,1739 | 3,1799 | 3,1867 |
| 94 | 3,2278 | 3,2346 | 3,2415 | 3,2484 | 3,2553 |
| 95 | 3,2968 | 3.3078 | 3.3107 | 3,3177 | 3,3246 |
| 96 | 3,3666 | 3,3736 | 3,3806 | 3,3877 | 3,3947 |
| 97 | 3,4371 | 3,4442 | 3,4513 | 3,4584 | 3,4655 |
| 98 | 3,5083 | 3,5155 | 3,5226 | 3,5297 | 3,5330 |
| 99 | 3,5803 | 3,5874 | 3,5948 | 3,6020 | 3,6093 |
| 100 | 3,6530 | 3,6603 | 3,6676 | 3,6749 | 3,6823 |

| 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| 1,9201 | 1,9254 | 1,9307 | 1,9360 | 1,9413 |
| 1,9734 | 1,9788 | 1,9342 | 1,9895 | 1,9949 |
| 2,0275 | 2,0329 | 2,0384 | 2,0438 | 2,0492 |
| 2,0822 | 2,0878 | 2,0922 | 2,0988 | 2,1041 |
| 2,1378 | 2,1434 | 2,1490 | 2,1546 | 2,1602 |
| 2,1940 | 2,1997 | 2,2057 | 2,2111 | 2,2168 |
| 2,2510 | 2,2568 | 2,2625 | 2,2682 | 2,2740 |
| 2,3088 | 2,3146 | 2,3204 | 2,3262 | 2,3320 |
| 2,3672 | 2,3731 | 2,3790 | 2,3849 | 2,3908 |
| 2,4264 | 2,4323 | 2,4382 | 2,4443 | 2,4502 |
| 2,4863 | 2,4923 | 2,4984 | 2,5044 | 2,5105 |
| 2,5469 | 2,5530 | 2,5591 | 2,5653 | 2,5714 |
| 2,6083 | 2,6145 | 2,6207 | 2,6269 | 2,6331 |
| 2,6704 | 2,6767 | 2,6829 | 2,6892 | 2,6945 |
| 2,7332 | 2,7396 | 2,7459 | 2,7522 | 2,7586 |
| 2,7968 | 2,8032 | 2,8096 | 2,8160 | 2,8224 |
| 2,8611 | 2,8676 | 2,8740 | 2,8805 | 2,8870 |
| 2,9261 | 2,9326 | 2,9392 | 2,9458 | 2,9523 |
| 2,9919 | 2,9985 | 3,0051 | 3,0117 | 3,0184 |
| 3,0583 | 3,0650 | 3,0717 | 3,0784 | 3,0852 |
| 3,1256 | 3,1323 | 3,1391 | 3,1459 | 3,1527 |
| 3,1935 | 3,2004 | 3,2072 | 3,2140 | 3,2209 |
| 3,2622 | 3,2691 | 3,2760 | 3,2820 | 3,2899 |
| 3,3316 | 3,3386 | 3,3456 | 3,3526 | 3,3596 |
| 3,4017 | 3,4088 | 3,4158 | 3,4229 | 3,4300 |
| 3,4726 | 3,4797 | 3,4869 | 3,4939 | 3,5012 |
| 3,5442 | 3,5514 | 3,5586 | 3,5658 | 3,5731 |
| 3,6165 | 3,6238 | 3,6311 | 3,6384 | 3,6457 |
| 3,6896 | 3,6969 | 3,7043 | 3,7117 | 3,7189 |

The

## The Deſcription and Uſe of the preceding Table.

In this *Table* are eleven Columns; in the firſt you have the Diameter of any Circle from 12 to 100 Inches, and in the ſecond ſtands the Area of that Circle.

### *EXAMPLE* I.

Let it be required to find the Area in Malt Buſhels of that Circle, whoſe Diameter is 36 Inches.

Find 36 in the firſt Column, and againſt that in the ſecond Column you'll find 0,4734, which ſignifies that that Cylinder whoſe Diameter is 36 Inches, will hold but ,4734 of a Buſhel of Malt upon an Inch.

### *EXAMPLE* II.

Let it be required to find the Area of that Circle, whoſe Diameter is 97 Inches.

Find 97 in the firſt Column, and againſt that in the ſecond you'll find 3,4371; the Area required.

### *EXAMPLE* III.

What's the Area of that Circle, whoſe Diameter is 150 Inches?

Find 15 in the firſt Column, and againſt that in the ſecond you'll find 0,0822, which is the Area of that Circle, whoſe Diameter is 15; but if you move the Comma two Places towards the Right-hand, 'twill be 008,22, *i.e.* 8,22; the Area required.

### *EXAMPLE* IV.

What's the Area of that Circle, whoſe Diameter is 42,6?

Find 42 Inches in the firſt Column, and ,6 at the Top; and in the Angle of Meeting you'll find 0,6629; the Area required.

### *EXAMPLE* V.

What's the Area of that Circle, whoſe Diameter is 148?

Find 14 in the firſt Column, and ,8 at the Top; then in the Angle of Meeting you'll find 0,0800; but ſetting the Point two Places back, 'twill be 008,00, *i.e.* 8,00; the Area required.

*EX-*

### EXAMPLE VI.

What's the Area of that Circle, whoſe Diameter is 148,5?

Find by the laſt Example, the Area of 149; which is 8,11, their Difference is ,11, and Half their Difference is ,055; which added to 8,00 is 8,055; the Area of 148,5 required.

### EXAMPLE VII.

What's the Area of that Circle, whoſe Diameter is 123,3?

1. Find the Area of 123, which is 5,52
2. Find the Area of 124 which is 5,61

their Difference is ,09; which multiplied by 3, gives 027; and that added to 5,52, makes 5,547; the Area required.

### EXAMPLE VIII.

What's the Area of that Circle, whoſe Diameter is 123,8?

1. Find the Area of 123, and that is 5,52
2. Find the Area of 124, and that is 5,61

their Difference is 0,09

which multiplied by 8, produceth ,072; and this added to 5,52 gives 5,592; the Area required.

By theſe eight Examples, it may ſufficiently appear, that the Area of any Circle from 12 to 100 Inches Diameter, may be found by Inſpection; and that the Area of any Circle fom 12 to 1000 Inches Diameter, may by the ſame *Tables* be found near enough in Practice.

## A TABLE to gauge right-lined Figures; as Squares, Triangles, Trapezia's, &c. giving the Content of the Squares from 21 to 120 Inches in Breadth, and from 30 to 310 in Length, by Inſpection; and conſequently of any other Length and Breadth, by Addition.

| 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|
| 30 | ,2930 | ,3069 | ,3209 | ,3349 | ,3488 |
| 40 | ,3907 | ,4093 | ,4279 | ,4465 | ,4651 |
| 50 | ,4884 | ,5116 | ,5448 | ,5581 | ,5813 |
| 60 | .5860 | ,6139 | ,6418 | ,6647 | ,6977 |
| 70 | ,6837 | ,7162 | ,7488 | ,7813 | ,8139 |
| 80 | ,7814 | ,8186 | ,8558 | ,8913 | .9302 |
| 90 | ,8790 | ,9209 | ,9627 | ,0046 | 1,0465 |
| 100 | .9767 | 1,0232 | 1,0697 | 1.1162 | 1,1628 |
| 110 | 1 0744 | 1,1256 | 1,1567 | 1,2279 | 1,2790 |
| 120 | 1.1720 | 1,2279 | 1,2837 | 1,3395 | 1,3953 |
| 130 | 1,2697 | 1,3302 | 3967 | 1,4511 | 1,5110 |
| 140 | 1 3674 | 1,4325 | 4976 | 1,5628 | 1 9279 |
| 150 | 1,4651 | 1,5349 | 1,6046 | 1,6744 | 1,7442 |
| 160 | 1,5628 | 1,6372 | 1,7116 | 1,7860 | 1,8604 |
| 170 | 1,6004 | 1,7395 | 1,8186 | 1,8970 | 1,9767 |
| 180 | 1,7580 | 1.8418 | 1,9255 | 2,0093 | 2,0930 |
| 190 | 1,8557 | 1,9422 | 2,0325 | 2,1209 | 2,2093 |
| 200 | 1.9534 | 2 0465 | 2 1395 | 2,2325 | 2,3256 |
| 210 | 2,0511 | 2,1488 | 2 2465 | 2,3442 | 2,4418 |
| 220 | 2,1488 | 2.2511 | 2,3534 | 2,4558 | 2,5581 |
| 230 | 2,2465 | 2,4535 | 2 4604 | 2,5674 | 2,6744 |
| 240 | 2,3442 | 2,5558 | 2,5674 | 2,6790 | 2,7907 |
| 250 | 2,4418 | 2,5581 | 2,6744 | 2,7906 | 2,9069 |
| 260 | 2,5395 | 2,6604 | 2,7831 | 2,9023 | 3.0232 |
| 270 | 2,6372 | 2,7628 | 2,8883 | 3,0139 | 3,1395 |
| 280 | 2,7348 | 2,8651 | 2,9953 | 3,1256 | 3,2558 |
| 290 | 2,8325 | 2,9674 | 3,1023 | 3,2372 | 3.3721 |
| 300 | 2,9302 | 2.0697 | 3 2093 | 3 3488 | 3,4884 |
| 1 | ,0097 | ,0102 | ,0107 | ,0111 | ,0116 |
| 2 | ,0195 | ,0204 | ,0214 | ,0223 | ,0232 |
| 3 | ,0293 | .0307 | ,0321 | ,0335 | ,0348 |
| 4 | 0390 | ,0390 | .0428 | ,0446 | ,0465 |
| 5 | ,0488 | 0488 | ,0535 | ,0558 | ,0581 |
| 6 | ,0586 | ,0586 | ,0642 | .0664 | ,0697 |
| 7 | ,0683 | ,0683 | 0749 | ,0781 | ,0813 |
| 8 | ,0781 | ,0781 | ,0856 | ,0893 | ,0930 |
| 9 | ,0879 | ,0879 | ,0963 | ,1004 | ,1046 |

| 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|
| .3626 | .3767 | .3907 | .4046 | .4186 |
| .4836 | .5023 | .5209 | .5395 | .5581 |
| .6046 | .6279 | .6511 | .6744 | .6976 |
| .7256 | .7535 | .7814 | .8093 | .8372 |
| .8465 | .8590 | .9116 | .9441 | .9767 |
| .9674 | 1.0046 | 1.0420 | 1.0790 | 1.1162 |
| 1.0883 | 1.1302 | 1.1721 | 1.2139 | 1.2558 |
| 1.2093 | 1.2558 | 1 3023 | 1.3488 | 1.3953 |
| 1.3302 | 1.3814 | 1.4325 | 1.4837 | 1.5349 |
| 1.4511 | 1.5069 | 1.5628 | 1.6186 | 1.6744 |
| 1.5721 | 1.6325 | 1.6930 | 1.7534 | 1.8139 |
| 1.6930 | 2.7581 | 1.8232 | 1.8884 | 1.9535 |
| 1.8139 | 1.8837 | 1.9534 | 2.0235 | 2.0930 |
| 1.9349 | 2.0090 | 2.0837 | 2.1581 | 2.2325 |
| 2.0358 | 2.1348 | 2.2139 | 2.2930 | 2.3721 |
| 2.1766 | 2.2604 | 2.3441 | 2.4279 | 2.5116 |
| 2.2976 | 2.3860 | 2.4744 | 2.5628 | 2.6511 |
| 2.4186 | 2.5116 | 2.6046 | 2.6976 | 2.7906 |
| 2.5395 | 2.6372 | 2.5348 | 2.8325 | 2.9302 |
| 2 6604 | 2.7628 | 2 8651 | 2.9674 | 3.0697 |
| 2.7814 | 2.8884 | 2.9953 | 3.1023 | 3.2093 |
| 2.9023 | 3 0139 | 2.1256 | 3.2372 | 3.3488 |
| 3.0232 | 3.1395 | 2.2558 | 3.3721 | 3.4883 |
| 3.1442 | 3.2651 | 3.3860 | 3.5069 | 3.6279 |
| 3.2651 | 3.3907 | 3.5163 | 3.6418 | 3.7674 |
| 3.3860 | 3.5163 | 3.6465 | 3.7767 | 3.9069 |
| 3 5069 | 3.6418 | 3.7767 | 3.9116 | 4.0465 |
| 3.6279 | 3.7674 | 3.9069 | 4.0465 | 4.1860 |
| 0120 | .0125 | .0130 | .0135 | 0139 |
| 0242 | .0251 | .0260 | .0269 | .0279 |
| 0362 | .0376 | 0390 | .0404 | .0418 |
| 0482 | .0502 | 0521 | .0539 | .0558 |
| .0604 | 0621 | .0651 | .0674 | .0697 |
| 0725 | .0753 | .0781 | .0809 | .0837 |
| 0846 | .0859 | .0911 | .0944 | .0976 |
| .0967 | .1004 | .1042 | .1079 | .1116 |
| .1088 | .1130 | .1172 | .1213 | .1255 |

| | 31 | 32 | 33 | 34 | 35 |
|---|---|---|---|---|---|
| 30 | .4325 | .4465 | .4604 | .4744 | .4883 |
| 40 | .5767 | .5953 | .6139 | .6325 | .6511 |
| 50 | .7209 | .7442 | .7674 | .7907 | .8139 |
| 60 | .8651 | 8930 | .9209 | .9488 | .9767 |
| 70 | 1.0093 | 1.0418 | 1.0744 | 1.1069 | 1.1395 |
| 80 | 1.1535 | 1.1907 | 1.2279 | 1.2651 | 1.3023 |
| 90 | 1.2976 | 1.3395 | 1.3814 | 1.4232 | 1.4651 |
| 100 | 1.4418 | 1.4883 | 1.5349 | 1.5813 | 1.6279 |
| 110 | 1.5860 | 1.6372 | 1.6883 | 1.7395 | 1.7907 |
| 120 | 1.7302 | 1.7860 | 1.8418 | 1.8977 | 1.9535 |
| 130 | 1.8744 | 1.9349 | 1.9953 | 2.0558 | 2.1162 |
| 140 | 2.0186 | 2.0837 | 2.1488 | 2.2139 | 2.2790 |
| 150 | 2.1628 | 2.2325 | 2.3023 | 2.3721 | 2.4418 |
| 160 | 2.3069 | 2.3814 | 2.4558 | 2.5023 | 2.6046 |
| 170 | 2.4511 | 2.5302 | 2.6093 | 2.6884 | 2.7674 |
| 180 | 2.5953 | 2.6790 | 2.7628 | 2.8465 | 2.9302 |
| 190 | 2.7395 | 2.8279 | 2.9162 | 3.0046 | 3.0930 |
| 200 | 2.8837 | 2.9767 | 3.0697 | 3.2628 | 3.2558 |
| 210 | 3.0279 | 3.1256 | 3.2232 | 3.3209 | 3.4186 |
| 220 | 3.1720 | 3.2744 | 3.3767 | 3.4791 | 3.5814 |
| 230 | 3.3162 | 3.4232 | 3.5302 | 3.6342 | 3.7442 |
| 240 | 3.4164 | 3.5721 | 3.6837 | 3.7923 | 3.8069 |
| 250 | 3.6046 | 3.7209 | 3.8372 | 3.9534 | 4.0697 |
| 260 | 3.7488 | 3.8697 | 3.9907 | 4.1116 | 4.2321 |
| 270 | 3.8930 | 4.0186 | 4.1442 | 4.3697 | 4.3952 |
| 280 | 4.0372 | 4.1674 | 4.2976 | 4.4279 | 4.5581 |
| 290 | 4.1814 | 4.3163 | 4.4516 | 4.5860 | 4.7200 |
| 300 | 4.3256 | 4.4651 | 4.6046 | 4.7442 | 4.8837 |
| 1 | 0141 | 0148 | 0153 | .0158 | 0162 |
| 2 | .0288 | 0297 | 0307 | .0316 | 0325 |
| 3 | .0432 | 0466 | 0460 | .0474 | 0488 |
| 4 | 0576 | 0595 | 0614 | .0632 | 0651 |
| 5 | .0720 | .0744 | 0767 | 0790 | .0813 |
| 6 | .0865 | .0893 | 0921 | 0948 | .0976 |
| 7 | .1009 | 1042 | .1074 | 1107 | .1139 |
| 8 | .1153 | .1190 | 1228 | 1265 | .1302 |
| 9 | .1297 | .1339 | .1381 | 1423 | .1465 |

Geo: Cole [illegible] this Book

| 36 | 37 | 38 | 39 | 40 |
|---|---|---|---|---|
| .5023 | .5163 | .5302 | .5442 | .5581 |
| .6697 | .6883 | .7069 | .7246 | .7441 |
| .8372 | .8604 | .8837 | .9069 | .9302 |
| 1.0046 | 1 0325 | 1 0604 | 1.0883 | 1.1163 |
| 1.1721 | 1.2024 | 1.2372 | 1.2097 | 1.2033 |
| 1.3395 | 1.3767 | 1.4139 | 1.4546 | 1.4884 |
| 1.5069 | 1.5488 | 1.5907 | 1.6325 | 1.6744 |
| 1.6744 | 1.7209 | 1.7674 | 1.8129 | 1.8604 |
| 1.8418 | 1.8930 | 1.9442 | 1.9953 | 2.0465 |
| 2 0093 | 2 0651 | 2.1209 | 2.1767 | 2.2325 |
| 2.1767 | 2.2372 | 2 2976 | 2.3581 | 2.4186 |
| 2.3442 | 2.4093 | 2 4744 | 2.5395 | 2.6046 |
| 2.5162 | 2.5814 | 2.6511 | 2.7209 | 2.7900 |
| 2.6791 | 2.7504 | 2.8279 | 2.9023 | 2.9767 |
| 2.8465 | 2.9255 | 3.0046 | 3 0837 | 3.1627 |
| 3 0139 | 2.9767 | 3 1814 | 3 2651 | 3.3488 |
| 3.1849 | 3.2697 | 3 3581 | 3.4465 | 3 5349 |
| 3 3488 | 3.4418 | 3 5348 | 3.6279 | 3.7209 |
| 3.5162 | 3.6139 | 3.7116 | 3.8093 | 3.9069 |
| 3.6837 | 3.7860 | 3 8883 | 3.9907 | 4 0930 |
| 3.8511 | 3.9581 | 4 0651 | 4 1721 | 4.2790 |
| 4.0186 | 4.1302 | 4.2418 | 4.3535 | 4.4465 |
| 4.1860 | 4.3023 | 4.4186 | 4 5348 | 4.6511 |
| 4.3535 | 4.4744 | 4 5953 | 4.7162 | 4.8372 |
| 4.5209 | 4.6465 | 4.9720 | 4.8976 | 5.0232 |
| 4.6883 | 4.8186 | 4.7488 | 5.0790 | 5 2093 |
| 4.8558 | 4.9906 | 5.1255 | 5.2604 | 5.3953 |
| 5.0232 | 5.1627 | 5.3023 | 5.4418 | 5.8112 |
| 0167 | 0172 | 0176 | .0181 | .0186 |
| .0335 | .0344 | 0353 | 0362 | 0372 |
| 0502 | .0516 | .0530 | .0544 | .0558 |
| 0669 | .0688 | .0707 | .0725 | 0744 |
| 0837 | .0860 | .0883 | .0907 | 0933 |
| .1004 | .1032 | .1060 | .1088 | .1116 |
| .1172 | .1202 | .1237 | .126[illegible] | .1202 |
| .1339 | .1376 | .1414 | .1454 | .1488 |
| .1507 | .1548 | 1590 | .1632 | .1674 |

| | 41 | 42 | 43 | 44 | 45 |
|---|---|---|---|---|---|
| 30 | .5720 | .5860 | .0000 | .6139 | .6279 |
| 40 | .7628 | .7814 | .8000 | .8186 | .8372 |
| 50 | .9535 | .9767 | 1.0000 | 1,0232 | 1.0455 |
| 60 | 1.1442 | 1.1721 | 1.2000 | 1 2270 | 1.2558 |
| 70 | 1.3348 | 1.3674 | 1.4001 | 1.4325 | 1.4651 |
| 80 | 1.5255 | 1.5628 | 1.6000 | 1.6372 | 1.6744 |
| 90 | 1.7162 | 1.7581 | 1.8000 | 1.8418 | 1.8857 |
| 100 | 1.9069 | 1.9535 | 2.0000 | 2.0465 | 2.0930 |
| 110 | 2.0976 | 2.1488 | 2.2000 | 2.2511 | 2.3023 |
| 120 | 2.2884 | 2.3442 | 2.4000 | 2.4558 | 2.5126 |
| 130 | 2 4790 | 2.5395 | 2.6000 | 2.6604 | 2.7209 |
| 140 | 2.6697 | 2.7348 | 2.8000 | 2.8651 | 2.9302 |
| 150 | 2.8604 | 2.9302 | 3.0000 | 3.0697 | 3.1395 |
| 160 | 3.0511 | 3.1255 | 3.2000 | 3.2744 | 3.3488 |
| 170 | 3.2418 | 3.3289 | 3.4000 | 3.4790 | 3.5581 |
| 180 | 3.4325 | 3.5162 | 3.6000 | 3 6837 | 3.7614 |
| 190 | 3.6232 | 3.7116 | 3.8000 | 3.8883 | 3.9767 |
| 200 | 3.8139 | 3.9069 | 4 0000 | 4.0930 | 4 1860 |
| 210 | 4.0046 | 4.1023 | 4.2000 | 4.2976 | 4 3953 |
| 220 | 4.1953 | 4.2976 | 4.4000 | 4.5023 | 4.6046 |
| 230 | 4.3860 | 4.4930 | 4.6000 | 4.7069 | 4.8139 |
| 240 | 4.5767 | 4.6883 | 4.8000 | 4 9116 | 5.0232 |
| 250 | 4.7674 | 4.8837 | 5.0000 | 5.1162 | 5.2325 |
| 260 | 4.9581 | 5.0700 | 5.2000 | 5.3209 | 5.4418 |
| 270 | 5.1488 | 5.2744 | 5.4000 | 5.5255 | 5.6511 |
| 280 | 5.3395 | 5.4697 | 5.6000 | 5.7347 | 5.8604 |
| 290 | 5.5302 | 5.6651 | 5 8000 | 5 9347 | 6.0697 |
| 300 | 5.7209 | 5 8604 | 6.0000 | 6.1394 | 6.2790 |
| 1 | .0190 | 0195 | 0200 | 0204 | .0209 |
| 2 | 0381 | 0390 | .0400 | 0409 | 0418 |
| 3 | .0572 | 0586 | 0600 | 0613 | .0628 |
| 4 | .0762 | 0781 | 0800 | 0818 | .0837 |
| 5 | .0953 | .0976 | .1000 | 1023 | .1046 |
| 6 | .1144 | .1172 | .1200 | 1228 | .1265 |
| 7 | .1335 | .1367 | 1400 | .1432 | .1465 |
| 8 | .1525 | .1562 | 1600 | .1637 | .1674 |
| 9 | .1716 | .1758 | .1800 | 1841 | .1883 |

| 46 | 47 | 48 | 49 | 50 |
|---|---|---|---|---|
| .6418 | .6558 | .6697 | .6837 | ,6976 |
| .8558 | .8744 | .8930 | .9116 | ,9302 |
| 1.0697 | 1.0930 | 1.1762 | 1.1395 | 1,1627 |
| 1.2837 | 1.3116 | 1.3395 | 1.3674 | 1,3953 |
| 1.4676 | 1.5302 | 1.5651 | 1.5953 | 1.6279 |
| 1.7116 | 1.7488 | 1.7860 | 1.8232 | 1.8604 |
| 1.9255 | 1.9674 | 2.0093 | 2.0511 | 2.0930 |
| 2.1395 | 2.1860 | 2.2325 | 2.2790 | 2.3255 |
| 2.3534 | 2.4046 | 2.4458 | 2.5069 | 2.5581 |
| 2.5674 | 2.6232 | 2.6790 | 2.7348 | 2.7906 |
| 2.7813 | 2.8418 | 2.9023 | 2.9618 | 3.0232 |
| 2.9553 | 3.0604 | 3.1255 | 3.1807 | 3.2558 |
| 3.2093 | 3.2790 | 3.3488 | 3.4186 | 3.4883 |
| 3.4232 | 3.4976 | 3.5720 | 3.6465 | 3.7269 |
| 3.6372 | 3.7162 | 3.7593 | 3.8744 | 3.9524 |
| 3.8511 | 3.9348 | 4.0186 | 4.1022 | 4.1860 |
| 4.0651 | 4.1534 | 4 2418 | 4.3202 | 4.4186 |
| 4.2790 | 4.3720 | 4.4651 | 4.5581 | 4.6511 |
| 4.4930 | 4 5906 | 4.6883 | 4.7860 | 4.8837 |
| 4.7069 | 4.8902 | 4.9116 | 5.0149 | 4.1162 |
| 5.9204 | 5 0279 | 5.1348 | 5.2419 | 5.3488 |
| 5.1348 | 5.2465 | 5.3581 | 5.4699 | 5.5813 |
| 5.3488 | 5.4651 | 5.5813 | 5.6976 | 5.8139 |
| 5.5627 | 5.6847 | 5.8046 | 5.9255 | 5.0465 |
| 5.7767 | 5 9055 | 6.0279 | 6.1534 | 6.2790 |
| 5.9906 | 6.1019 | 6.2511 | 6.3814 | 6.5116 |
| 6.2046 | 6.3305 | 6.4744 | 6.6093 | 6.7441 |
| 6.4186 | 6.5581 | 6.6976 | 6.8372 | 6.9767 |
| 0214 | 0 218 | .0223 | .0228 | .0232 |
| .0427 | 0.437 | .0446 | .0455 | .0465 |
| .0641 | 0.655 | .0669 | 0683 | .0697 |
| .0855 | 0.874 | .0893 | .0911 | 0930 |
| .1069 | .1093 | .1176 | .1139 |  |
| .1283 | .1311 | .1339 | .1367 | .1162 |
| .1467 | .1530 | .1565 | .1595 | .1395 |
| .1711 | .1748 | .1786 | .1823 | .1628 |
| .1925 | .1967 | .2000 | .2051 | .1860 |

| | 51 | 52 | 53 | 54 | 55 |
|---|---|---|---|---|---|
| 30 | ,7116 | ,7255 | ,7395 | ,7534 | ,7674 |
| 40 | ,9488 | ,9674 | ,9860 | 1,0046 | 1,0232 |
| 50 | 1,1862 | 1,2093 | 1,2325 | 1,2558 | 1,2790 |
| 60 | 1,4232 | 1,4511 | 1,4790 | 1.4069 | 1,5348 |
| 70 | 1 6604 | 1,6930 | 1,7255 | 1,7581 | 1,7906 |
| 80 | 1,8976 | 1,9348 | 1,9720 | 1,9093 | 2,0465 |
| 90 | 2,1648 | 2,1767 | 2,2180 | 2,2604 | 2,3023 |
| 100 | 2,3720 | 2.4186 | 2.4651 | 2,5116 | 2,5581 |
| 110 | 2,6093 | 2,6604 | 2,7116 | 2,7627 | 2,8139 |
| 120 | 2,8465 | 2,9023 | 2,9581 | 3,0139 | 3,0697 |
| 130 | 3,0837 | 3,1441 | 3,2046 | 3.2651 | 3,3255 |
| 140 | 3.3209 | 3,3860 | 3,4511 | 3,5162 | 3,5813 |
| 150 | 3,5581 | 3.6279 | 3.6976 | 3,7674 | 3,8372 |
| 160 | 3,7953 | 3,8697 | 3,9441 | 4,0186 | 4,0930 |
| 170 | 4,0325 | 4,1116 | 4,1906 | 4,2697 | 4.3488 |
| 180 | 4,2697 | 4,3534 | 4.4372 | 4,5209 | 4.6406 |
| 190 | 4,5069 | 4,5953 | 4,6837 | 4,7720 | 4,8612 |
| 200 | 4 7441 | 4,8372 | 4,9302 | 5,0232 | 5,1162 |
| 210 | 4 9813 | 5,0790 | 5,1767 | 5,2740 | 5,3720 |
| 220 | 5,2186 | 5,3299 | 5,4232 | 5,5255 | 5 6279 |
| 230 | 5,4558 | 5,5627 | 5,6697 | 5,7767 | 5,8837 |
| 240 | 5,6930 | 5,8046 | 5,9162 | 6,0249 | 6,1495 |
| 250 | 5,6302 | 6,0465 | 6,1627 | 6,2790 | 6,3953 |
| 260 | 6,1674 | 6,2883 | 6,4093 | 6,5302 | 6,051 |
| 270 | 6,4046 | 6,5302 | 6,6558 | 6,7813 | 6,9069 |
| 280 | 6,6418 | 6,7720 | 6,9023 | 7,0325 | 7,1627 |
| 290 | 6,8790 | 7,0139 | 7,1488 | 7,2837 | 7,4180 |
| 300 | 7.1162 | 7,2558 | 7,3953 | 7,5348 | 7.6744 |
| 1 | ,0234 | ,0242 | 0246 | 0251 | ,0255 |
| 2 | 0474 | ,0483 | .0493 | ,0502 | ,0511 |
| 3 | 0711 | ,0725 | 0739 | ,0753 | ,0767 |
| 4 | .0948 | .0967 | 0986 | ,1004 | ,1023 |
| 5 | ,1186 | ,1209 | 1232 | .1255 | ,1279 |
| 6 | ,1423 | ,1451 | ,1479 | 1407 | ,1535 |
| 7 | ,1660 | ,1693 | .1725 | ,1758 | ,1790 |
| 8 | ,1897 | .1934 | ,1972 | ,1909 | ,2046 |
| 0 | ,2165 | .2176 | .2218 | ,2260 | ,2302 |

| 56 | 57 | 58 | 59 | 60 |
|---|---|---|---|---|
| .7813 | .7953 | .8093 | .8232 | .8372 |
| 1.0418 | 1.0604 | 1.0790 | 1.0976 | 1.1162 |
| 1.3023 | 1.3255 | 1.3488 | 1.3720 | 1.3953 |
| 1.5627 | 1.5906 | 1.6186 | 1.6485 | 1.6744 |
| 1.8237 | 1.8558 | 1.8883 | 1.9209 | 1.9534 |
| 2.0837 | 2.1209 | 2.1581 | 2.1953 | 2.2325 |
| 2.3441 | 2.3860 | 2.4279 | 2.4697 | 2.5116 |
| 2.6046 | 2.6511 | 2.6976 | 2.7441 | 2.7906 |
| 2.8651 | 2.9162 | 2.9674 | 3.0186 | 3.6097 |
| 3.1255 | 3.1813 | 3.2372 | 3.2930 | 3 3488 |
| 3.3860 | 3.4465 | 3.5069 | 3.5674 | 3.6279 |
| 3.6465 | 3.7116 | 3.7767 | 3.8418 | 3.9069 |
| 3.9069 | 3.9767 | 4.0465 | 4.1162 | 4.1860 |
| 4.1674 | 4.2418 | 4.3162 | 4.3906 | 4.4651 |
| 4.4279 | 4.5069 | 4.5860 | 4.6651 | 4.7441 |
| 4.6883 | 4.7720 | 4.8558 | 4.9395 | 5 0232 |
| 4.9488 | 5.0372 | 5.1255 | 5 2139 | 5.3023 |
| 5.2093 | 5.3023 | 5.3953 | 5.4883 | 5.5813 |
| 5.4697 | 5.5674 | 5.6651 | 5.7627 | 5 8604 |
| 5.7212 | 5.8325 | 5.9348 | 6.0372 | 6.1395 |
| 5.9816 | 6.0976 | 6.2046 | 6.3116 | 6.4186 |
| 6.2521 | 6.3627 | 6.4744 | 6.5860 | 6.6976 |
| 6.5126 | 6.6279 | 6.7441 | 6.8604 | 6.9767 |
| 6.7730 | 6.8930 | 7.0149 | 7.1348 | 7.3558 |
| 7.0330 | 7.1581 | 7.2887 | 7.4093 | 7.5348 |
| 7.2930 | 7.4232 | 7.5534 | 7.6837 | 7.8139 |
| 7.5534 | 7.6883 | 7.8232 | 7.9581 | 8.0929 |
| 7.8139 | 7.9534 | 8.0930 | 8.2325 | 8.3720 |
| .0260 | .0265 | .0269 | .0274 | .0279 |
| .0509 | .0530 | .0539 | .0548 | .0558 |
| .0781 | .0795 | .0809 | .0823 | .0837 |
| .1042 | .1060 | .1079 | .1097 | .1116 |
| .1302 | .1325 | .1348 | .1372 | .1395 |
| .1561 | .1590 | .1618 | .1648 | .1674 |
| .1823 | .1855 | .1888 | .1921 | .1953 |
| .2083 | .2120 | .2158 | .2195 | .2232 |
| .2344 | .2386 | .2328 | .2469 | .2511 |

| | 61 | 62 | 63 | 64 | 65 |
|---|---|---|---|---|---|
| 30 | .8511 | .8651 | .8790 | .8930 | .9069 |
| 40 | 1.1348 | 1.1534 | 1.1720 | 1.1906 | 1.2093 |
| 50 | 1.4186 | 1.4418 | 1.4651 | 1.4883 | 1.5116 |
| 60 | 1.7023 | 1.7302 | 1.7581 | 1.7860 | 1.8139 |
| 70 | 1.9860 | 2.0186 | 2.0511 | 2.0837 | 2.1162 |
| 80 | 2.2697 | 2.3069 | 2.3441 | 2.3813 | 2.4186 |
| 90 | 2.5534 | 2.5953 | 2.6372 | 2.6790 | 2.7209 |
| 100 | 2.8372 | 2.8837 | 2.9302 | 2.9767 | 2.0232 |
| 110 | 3.1209 | 3.1720 | 3.2232 | 3.2744 | 3.3255 |
| 120 | 3.4046 | 3.4804 | 3.5167 | 3.5720 | 3.6279 |
| 130 | 3.6883 | 3.7488 | 3.8096 | 3.8697 | 3.9302 |
| 140 | 3.9720 | 4.0372 | 4.1023 | 4.1674 | 4.2325 |
| 150 | 4.2558 | 4.3255 | 4.3953 | 4.4651 | 4.5348 |
| 160 | 4.5395 | 4.6139 | 4.6883 | 4.7629 | 4.8372 |
| 170 | 4.8232 | 4.9023 | 4.9813 | 5.0604 | 5.1395 |
| 180 | 5.1869 | 5.1906 | 5.2744 | 5.3581 | 5.4418 |
| 190 | 5.3906 | 5.4790 | 5.5674 | 5.6558 | 5.7465 |
| 200 | 5.6740 | 5.7641 | 5.8604 | 5.9534 | 6.0465 |
| 210 | 5.9581 | 6.0558 | 6.1534 | 6.2511 | 6.3488 |
| 220 | 6.2418 | 6.3441 | 6.4465 | 6.5488 | 6.6511 |
| 230 | 6.5255 | 6.6325 | 6.7395 | 6.8466 | 6.9534 |
| 240 | 6.8098 | 6.9209 | 7.0355 | 7.1441 | 7.2548 |
| 250 | 7.0930 | 7.2093 | 7.3255 | 7.4418 | 7.5577 |
| 260 | 7.3767 | 7.4976 | 7.6186 | 7.7395 | 7.8594 |
| 270 | 7.6604 | 7.7860 | 7.9116 | 8.0372 | 8.1627 |
| 280 | 7.9441 | 8.0744 | 8.2046 | 8.3348 | 8.4651 |
| 290 | 8.2279 | 8.3627 | 8.4976 | 8.6325 | 8.7674 |
| 300 | 8.5116 | 8.6511 | 8.7906 | 8 9302 | 8.0697 |
| 1 | 0283 | .0883 | .0293 | .0297 | .0202 |
| 2 | .0567 | .0576 | .0586 | .0595 | .0604 |
| 3 | .0851 | .0865 | 0879 | 0893 | .0907 |
| 4 | .1135 | .1153 | 1172 | .1190 | 1209 |
| 5 | .1418 | .1442 | 1465 | .1488 | .1511 |
| 6 | .1702 | .1730 | .1758 | .1786 | .1814 |
| 7 | .1986 | .2018 | .2051 | .2083 | .2116 |
| 8 | .2269 | .2307 | .2344 | .2381 | .2418 |
| 9 | .2553 | .2505 | .2637 | .2679 | .2720 |

130

| 66 | 67 | 68 | 69 | 70 |
|---|---|---|---|---|
| ,9209 | ,9348 | ,9488 | ,9627 | ,9767 |
| 1,2279 | 1,2465 | 1,2651 | 1,2837 | 1,3023 |
| 1,5348 | 1,5581 | 1,5813 | 1,6046 | 1,6279 |
| 1,8418 | 1,8697 | 1.8976 | 1,9255 | 1.9534 |
| 2,1488 | 2,1813 | 2,2189 | 2.2465 | 2,2790 |
| 2,4558 | 2,4930 | 2,5302 | 2,5674 | 2,6046 |
| 2,7627 | 2,8046 | 2,8465 | 2,8883 | 2,9032 |
| 3.0697 | 3,1162 | 3,1627 | 3,2093 | 3,2558 |
| 3,3767 | 3.4279 | 3,4790 | 3,5302 | 3,5813 |
| 3,6837 | 3.7395 | 3,7953 | 3,8511 | 3,9069 |
| 3.9906 | 4,0511 | 4,1116 | 4,1720 | 4,2325 |
| 4,2976 | 4.3627 | 4,4278 | 4,4930 | 4.5587 |
| 4,6046 | 4,6744 | 4,7440 | 4,8139 | 4,8837 |
| 4,9116 | 4.9860 | 5,0604 | 5,1348 | 5,2903 |
| 5,2186 | 5,2976 | 5,3767 | 5,4558 | 5.5348 |
| 5,5255 | 5,6093 | 5,6930 | 5,7767 | 5,8664 |
| 5,8325 | 5,9209 | 6,0099 | 6,0976 | 6,1860 |
| 6,1395 | 6,2325 | 6,3225 | 6,4185 | 6,5116 |
| 6,4465 | 6,5441 | 6,6418 | 6,7395 | 6,8372 |
| 6,7534 | 6.8558 | 6 9581 | 7,0604 | 7,1627 |
| 7,0604 | 7,1674 | 7,2744 | 7,3813 | 7,4883 |
| 7,3674 | 7,4790 | 7,5906 | 7,7023 | 7,8139 |
| 7,6744 | 7,7906 | 7,9069 | 8,0232 | 8,1295 |
| 7,9813 | 8,1023 | 8,2232 | 8,3441 | 8,4653 |
| 8,2883 | 8,4139 | 8,5395 | 8,6651 | 8,7906 |
| 8,5953 | 8,7255 | 8,8550 | 8,9860 | 9,1162 |
| 8,9023 | 9,0327 | 9,1720 | 9,0369 | 9,4418 |
| 9,2093 | 9,3488 | 9,4883 | 9,6279 | 9.7674 |
| ,0307 | ,0311 | ,0316 | ,0320 | ,0325 |
| ,0614 | ,0623 | ,0632 | ,0641 | 0651 |
| ,0920 | ,0935 | ,0948 | ,0962 | .0976 |
| ,1228 | ,1246 | .1265 | ,1283 | ,1302 |
| ,1535 | ,1558 | ,1581 | ,1604 | .1628 |
| ,1842 | ,1869 | ,1897 | ,1925 | .1953 |
| ,2148 | ,2181 | ,2218 | ,2246 | .2279 |
| ,2455 | ,2493 | ,2530 | ,2567 | ,2604 |
| ,2762 | ,2846 | ,2804 | ,2888 | ,2903 |

|  | 71 | 72 | 73 | 74 | 75 |
|---|---|---|---|---|---|
| 30 | ,9906 | 1,0046 | 1,0186 | 1,0325 | 1,0465 |
| 40 | 1,3209 | 1,3395 | 1,3581 | 1,3767 | 1,3953 |
| 50 | 1,6511 | 1,6744 | 1,6976 | 1,7209 | 1,7441 |
| 60 | 1,9813 | 2,0093 | 2,0372 | 2,0651 | 2,0930 |
| 70 | 2,3116 | 2,3441 | 2,3567 | 2,4063 | 2,4418 |
| 80 | 2,6418 | 2,6790 | 2,7162 | 2,7534 | 2,7906 |
| 90 | 2,9720 | 2,0139 | 3.0558 | 3,0976 | 3,1395 |
| 100 | 3 3023 | 3,3488 | 3.3953 | 3,4418 | 3,4883 |
| 110 | 3,6325 | 3,6837 | 3,7348 | 3,7860 | 3,8372 |
| 120 | 3,9627 | 4,0186 | 4,0744 | 4,1302 | 4,1861 |
| 130 | 4 2930 | 4,3534 | 4.4139 | 4.4744 | 4.5348 |
| 140 | 4,6232 | 4,6833 | 4.7534 | 4,8186 | 4,8837 |
| 150 | 4,9534 | 5,2032 | 5,0930 | 5,1627 | 5,2325 |
| 160 | 5,2837 | 5,3581 | 5 4325 | 5,5069 | 5.5813 |
| 170 | 5,6139 | 5.5930 | 5,7720 | 5,8511 | 4,9301 |
| 180 | 5,9441 | 6,0279 | 6,1116 | 6,1953 | 6,2790 |
| 190 | 6.2744 | 6,3622 | 6 4511 | 6,5395 | 6.6279 |
| 200 | 6,6046 | 6,6976 | 6,7906 | 6,8837 | 6,9767 |
| 210 | 6 9348 | 7,0325 | 7,1302 | 7.2790 | 7,3255 |
| 220 | 7,2651 | 7,3674 | 7,4897 | 7,5620 | 7,6744 |
| 230 | 7.5953 | 7,7023 | 7,8093 | 7,9162 | 8 0232 |
| 240 | 7,9255 | 8,0372 | 8,1488 | 8,2604 | 8,3720 |
| 250 | 8,2558 | 8,3720 | 8,4883 | 8,6046 | 8,7209 |
| 260 | 8,5860 | 8,7069 | 8,8279 | 8,9488 | 9,0697 |
| 270 | 8,9162 | 9,0418 | 9.1674 | 9,2930 | 9,4186 |
| 280 | 9,2465 | 9,3767 | 9.5069 | 9,6372 | 9,7674 |
| 290 | 9,5767 | 9,7116 | 9.8465 | 9,9813 | 10,1162 |
| 300 | 0,9065 | 10,046 | 10.1860 | 10,3255 | 10,4651 |
| 1 | ,0330 | 0334 | 0339 | ,0344 | 0348 |
| 2 | ,0660 | ,0669 | ,0679 | ,0688 | ,0697 |
| 3 | ,0990 | .1004 | ,1018 | ,0032 | .1046 |
| 4 | ,1321 | ,1339 | .1358 | ,1376 | ,1395 |
| 5 | ,1651 | ,1674 | .1697 | ,1721 | ,1744 |
| 6 | ,1981 | ,2009 | ,2037 | ,2065 | .2093 |
| 7 | .2311 | ,2344 | ,2356 | ,2406 | .2442 |
| 8 | ,2642 | ,2679 | ,2716 | ,2753 | .2790 |
| 9 | .2972 | ,3014 | .3056 | .3097 | ,3139 |

| 76 | 77 | 78 | 79 | 80 |
|---|---|---|---|---|
| 1,0604 | 1,0744 | 1,0883 | 1,1023 | 1,1162 |
| 1,4139 | 1,4325 | 1,4511 | 1,4697 | 1,4883 |
| 1,7674 | 1,7906 | 1,8139 | 1,8372 | 1,8604 |
| 2,1209 | 2,1488 | 2,1767 | 2,6046 | 2.2225 |
| 1,4744 | 2,5069 | 2,5395 | 2,5720 | 2,6046 |
| 1,8279 | 2,8651 | 2,9023 | 2,9395 | 2,9767 |
| 3,1813 | 3,2232 | 3,2651 | 3,3069 | 3,3488 |
| 3,5348 | 3,5813 | 3.6279 | 3.6744 | 3,7209 |
| 3,8883 | 3,9395 | 3.9906 | 4,0418 | 4,0930 |
| 4,2418 | 4,3976 | 4,3534 | 4,4093 | 4,4657 |
| 4,5953 | 4,6558 | 4,7162 | 4,7767 | 4,8372 |
| 4,9488 | 5,0139 | 5.0790 | 5.1441 | 5,2093 |
| 5,3023 | 5,3720 | 5,4418 | 5,5116 | 5,5813 |
| 5,6558 | 5.7302 | 5,8046 | 5,8790 | 5,9534 |
| 6,0093 | 6,0883 | 6,1694 | 6,2465 | 6,3255 |
| 6,3627 | 6,4465 | 6.5302 | 6,6139 | 6.6976 |
| 6,7162 | 6,8046 | 6,8930 | 6,9813 | 7,0697 |
| 7,0697 | 7,1727 | 7,2558 | 7,3488 | 7,4418 |
| 7,4232 | 7,5209 | 7,6186 | 7,7162 | 7.8139 |
| 7,7767 | 7,8790 | 7,9813 | 8,0837 | 8,1860 |
| 8,1302 | 8,2371 | 8,3441 | 8,4511 | 8,5581 |
| 8,4837 | 8,6953 | 8,7069 | 8,8186 | 8,9302 |
| 8,8372 | 8,9534 | 9,0697 | 9,1860 | 9.3023 |
| 9.1906 | 9,3116 | 9.4325 | 9,5524 | 9.6744 |
| 9.5441 | 9,6697 | 9.7935 | 9,9209 | 10,0465 |
| 9,8976 | 10,0278 | 10,1581 | 10,2883 | 10,4186 |
| 10,2511 | 10,3860 | 10.5109 | 10,6558 | 10,7906 |
| 10,6046 | 10.7441 | 10,8837 | 10.3032 | 11,1627 |
| .0353 | 0358 | ,0362 | ,0367 | ,0372 |
| ,0707 | ,0717 | ,0725 | ,0734 | ,0744 |
| ,1060 | .1074 | ,1088 | ,1102 | ,1116 |
| ,1414 | .1432 | ,1451 | ,1469 | ,1488 |
| ,1767 | ,1790 | ,1834 | ,1837 | ,1860 |
| ,2121 | ,2148 | ,2176 | ,2204 | ,2232 |
| ,2474 | ,2506 | ,2539 | ,2572 | ,2604 |
| ,2828 | ,2865 | ,2902 | ,2939 | ,2976 |
| ,3181 | ,3223 | ,3265 | .3307 | ,3248 |

| | 81 | 82 | 83 | 84 | 85 |
|---|---|---|---|---|---|
| 30 | 1.1302 | 1.1441 | 1.1581 | 1.1720 | 1.1860 |
| 40 | 1.5069 | 1.5255 | 1.5441 | 1.5627 | 1.5813 |
| 50 | 1.8837 | 1.9069 | 1.9302 | 1.9534 | 1.9767 |
| 60 | 2.2604 | 2.2882 | 2.3167 | 2.3441 | 2.3720 |
| 70 | 2.6372 | 2.6697 | 2.7023 | 2.7348 | 2.7674 |
| 80 | 3.0139 | 3.0511 | 3.0883 | 3.1255 | 3.1627 |
| 90 | 3.3906 | 3.4325 | 3.4741 | 3.5162 | 3.5581 |
| 100 | 3.7674 | 3.8139 | 3.8604 | 3.9069 | 3.9534 |
| 110 | 4.1441 | 4.1953 | 4.2465 | 4.2976 | 4.3488 |
| 120 | 4.5209 | 4.5167 | 4.6325 | 4.6883 | 4.7441 |
| 130 | 4.8976 | 4.9581 | 5.0186 | 4.0704 | 5.1395 |
| 140 | 5.2744 | 5.3295 | 5.4046 | 5.4697 | 5.5348 |
| 150 | 5.6511 | 5.7209 | 5.7906 | 5.8604 | 5.6302 |
| 160 | 6.0279 | 6.1023 | 6.1767 | 6.2511 | 6.3255 |
| 170 | 6.4046 | 6.4837 | 6.5627 | 6.6418 | 6.7209 |
| 180 | 6.7813 | 6.8651 | 6.9488 | 7.0325 | 7.1162 |
| 190 | 7.1581 | 7.2465 | 7.3347 | 7.4232 | 7.5116 |
| 200 | 7.5348 | 7.6279 | 7.7209 | 7.8139 | 7.9069 |
| 210 | 7.9116 | 8.0093 | 8.1069 | 8.2046 | 8.3023 |
| 220 | 8.2883 | 8.3906 | 8.4920 | 8.5953 | 8.6976 |
| 230 | 8.6651 | 8.7720 | 8.8790 | 8.9860 | 9.0930 |
| 240 | 9.0413 | 9.1534 | 9.2651 | 9.3767 | 9.4883 |
| 250 | 9.4186 | 9.5348 | 9.6511 | 9.7674 | 9.8837 |
| 260 | 9.7953 | 9.9162 | 10.0372 | 10.1581 | 10.2793 |
| 270 | 10.1720 | 10.2976 | 10.4232 | 10.5488 | 10.6744 |
| 280 | 10.5488 | 10.6790 | 10.8093 | 10.9395 | 11.0697 |
| 290 | 10.9255 | 10.0604 | 11.1953 | 11.3303 | 11.4651 |
| 300 | 10.3023 | 10.4118 | 11.5813 | 11.7209 | 11.8604 |
| 1 | .0376 | .0381 | .0386 | .0390 | .0395 |
| 2 | .0753 | .0761 | .0772 | .0781 | .0790 |
| 3 | .1130 | .1144 | .1158 | .1172 | .1186 |
| 4 | .1507 | .1525 | .1544 | .1562 | .1581 |
| 5 | .1883 | .1900 | .1930 | .1953 | .1976 |
| 6 | .2260 | .2288 | .2316 | .2344 | .2372 |
| 7 | .2637 | .2669 | .2702 | 2735 | .2769 |
| 8 | .3014 | .3051 | .3088 | .3125 | .3162 |
| 9 | .3390 | .3432 | .3474 | 3516 | .3558 |

| 86 | 87 | 88 | 89 | 90 |
|---|---|---|---|---|
| 1.2000 | 1.2139 | 1.2279 | 1.2418 | 1.2558 |
| 1.6000 | 1.6189 | 1.7372 | 1.6558 | 1.6744 |
| 2.0000 | 2.0232 | 2.0465 | 2.0697 | 2.0930 |
| 2.8000 | 2.4279 | 2.4558 | 2.4837 | 2.5116 |
| 2.8000 | 2.8325 | 2.8852 | 2.8676 | 2.9302 |
| 3.2000 | 3.2372 | 3.2744 | 3.3116 | 3.3488 |
| 3.6000 | 3.6418 | 3.6837 | 3.7255 | 3.7674 |
| 4.0000 | 4.0465 | 3.0930 | 4.1395 | 4.1860 |
| 4.4000 | 4.4511 | 4.5023 | 4.5534 | 4.6046 |
| 4.8000 | 4.8558 | 4.9116 | 4.9674 | 5.0232 |
| 5.2000 | 5.1604 | 5.3209 | 5.3813 | 5.4418 |
| 5.6000 | 5.9651 | 5.7302 | 5.7953 | 5.8604 |
| 6.0000 | 6.0679 | 6.1395 | 6.2093 | 6.2790 |
| 6.4000 | 6.4744 | 6.5488 | 6.6232 | 6.6976 |
| 6 7000 | 6.8760 | 6.9581 | 7.0372 | 7.1162 |
| 7.2000 | 7.2837 | 7.2974 | 7.4511 | 7.5348 |
| 7.6000 | 7.6883 | 7.7767 | 7.8651 | 7.9534 |
| 8.0000 | 8.0930 | 8.1860 | 8.2790 | 8.3720 |
| 8.4000 | 8.4978 | 8.5953 | 8.6930 | 8.7906 |
| 8.8000 | 8.9023 | 9.0046 | 9.0069 | 9.2093 |
| 9.2000 | 9.3069 | 9.4139 | 9.5209 | 9.6279 |
| 9.6000 | 9.7016 | 9.8232 | 9.9348 | 10.0465 |
| 10.0000 | 10.1162 | 10.2325 | 10.3488 | 10.4652 |
| 10.4000 | 10.5209 | 10.6418 | 10.7627 | 10.8837 |
| 10.8000 | 10.9255 | 11.0511 | 11.1767 | 11.3023 |
| 11.2000 | 11.3302 | 11,4604 | 11.5906 | 11.7209 |
| 11.6000 | 11.7348 | 11.8697 | 11.9046 | 12.1395 |
| 12.0000 | 12.1395 | 12.2790 | 12.4186 | 12.5581 |
| .0400 | .0404 | .0409 | .0413 | .0418 |
| .0800 | .0809 | .0818 | .0828 | .0800 |
| .1200 | .1214 | .1227 | .1242 | .1255 |
| .1600 | .1619 | .1737 | .1455 | .1674 |
| .2000 | .2023 | .2046 | .2006 | .2093 |
| .2400 | .2428 | .2455 | .2483 | 2511 |
| .2800 | .2832 | .2865 | .2897 | .2930 |
| .3200 | .3237 | .3274 | .3311 | 3348 |
| .3600 | .3642 | .3683 | .3725 | .3767 |

| | 91 | 92 | 93 | 94 | 95 |
|---|---|---|---|---|---|
| 30 | 1.2695 | 1.2837 | 1.2976 | 1.3116 | 1.3255 |
| 40 | 1.693 | 1.7171 | 1.7302 | 1.7488 | 1.7674 |
| 50 | 2.1162 | 2.1395 | 2.1627 | 2.1862 | 2.2093 |
| 60 | 2.5395 | 2.5674 | 2 5953 | 2.6232 | 2.6511 |
| 70 | 2.9627 | 2.9953 | 3.0279 | 3.0604 | 3.0930 |
| 80 | 3.3860 | 3.4232 | 3.4604 | 3.4970 | 3.5348 |
| 90 | 3.8093 | 3.8512 | 3.8930 | 3.9348 | 3.9767 |
| 100 | 4.2325 | 4.2760 | 4.3255 | 4.3720 | 4.4186 |
| 110 | 4.6558 | 4.7009 | 4.7581 | 4.8093 | 4.8604 |
| 120 | 5.079 | 5.1748 | 5.1906 | 5.2465 | 5.3023 |
| 130 | 5.5023 | 5.5627 | 5.6232 | 5.6830 | 5.7441 |
| 140 | 5.9255 | 5.9906 | 6.0558 | 6.1209 | 6.1860 |
| 150 | 6.3488 | 6.4186 | 6.4883 | [illegible] | 6.6279 |
| 160 | 6.7720 | 6.8465 | 6.9209 | [illegible] | [illegible] |
| 170 | 7.1953 | 7.2741 | 7.3534 | [illegible] | [illegible] |
| 180 | 7.6186 | 7 7022 | 7.786 | [illegible] | [illegible] |
| 190 | 8.0418 | 8.1202 | [illegible] | [illegible] | [illegible] |
| 200 | 8.4651 | 8.5813 | [illegible] | [illegible] | [illegible] |
| 210 | 8.8883 | 8.9860 | 9.0837 | [illegible] | [illegible] |
| 220 | 9.3116 | 9.4129 | 9.5162 | [illegible] | [illegible] |
| 230 | 9.7348 | 9.4818 | 9.9488 | [illegible] | [illegible] |
| 240 | 10.1581 | 10.2697 | 10.3813 | [illegible] | [illegible] |
| 350 | 10.5813 | 10.6976 | 10.8135 | [illegible] | [illegible] |
| 260 | 11.0047 | 11.1125 | 11.2465 | [illegible] | [illegible] |
| 270 | 11.4279 | 11.5534 | 11.6790 | [illegible] | [illegible] |
| 280 | 11.8511 | 11.9813 | 12.1106 | [illegible] | [illegible] |
| 290 | 12.2544 | 11.4093 | 12.5444 | [illegible] | [illegible] |
| 300 | 12.6976 | 12.8272 | 12 9767 | [illegible] | [illegible] |
| 1 | .0423 | .0427 | .0432 | .0437 | [illegible] |
| 2 | .0846 | .0858 | 0865 | .0874 | [illegible] |
| 3 | .1269 | .1283 | .1297 | .1311 | 1325 |
| 4 | .1693 | .1717 | .1730 | 1748 | 1767 |
| 5 | .2116 | 2139 | 2162 | .2186 | .2209 |
| 6 | .2539 | 2567 | .2595 | .2623 | .2651 |
| 7 | .2962 | .2995 | .3028 | .3060 | 3093 |
| 8 | .3386 | 3423 | 3460 | .3497 | .3534 |
| 9 | .3809 | 3851 | .3893 | .3924 | 3976 |

| 96 | 97 | 98 | 99 | 100 |
|---|---|---|---|---|
| 1.3395 | 1.3534 | 1.3674 | 1.3813 | 1 3953 |
| 1.7860 | 1.8046 | 1.8232 | 1.8418 | 1.8604 |
| 2 2325 | 2.2558 | 2.2790 | 2.3023 | 2.3258 |
| 2.6790 | 2.7069 | 2 7348 | 2.7627 | 2.7907 |
| 3.1255 | 3.1581 | 3.1900 | 3.2232 | 3.2558 |
| 3.5720 | 3.6093 | 3.6465 | 3.6837 | 3 7209 |
| 4 0186 | 4.0604 | 4.1023 | 4.1441 | 4.1860 |
| 3.4651 | 4 5116 | 4 5581 | 4 6046 | 4 6511 |
| 4.9110 | 4.9627 | 5 0139 | 5.0651 | 5.1162 |
| 5.3581 | 5.4139 | 5.4697 | 5.5259 | 5.5813 |
| 5.8046 | 5.865[illegible] | 5 9255 | 5.9860 | 6.0465 |
| 6 2511 | 6.3162 | 6.3813 | 6 4465 | 6.5116 |
| 6.6976 | 6.7674 | 6 8372 | 6 9069 | 6.9767 |
| 7.1441 | 7.2186 | 7.2930 | 7.3674 | 7.4418 |
| 7 5906 | 7 6697 | 7.7488 | 7.8279 | 7 9069 |
| 8.0372 | 8.1209 | 8.2046 | 8.2886 | 8.3720 |
| 8 4837 | 8.5720 | 8.0004 | 8.7488 | 8.8372 |
| 8.9302 | 9.0232 | 9.1162 | 9.2993 | 9.3023 |
| 9.3767 | 9.4744 | 9.5720 | 9.6679 | 9.7674 |
| 9.8232 | 9 9255 | 10 0279 | 10.1302 | 10.2325 |
| 10 2697 | 10.3767 | 10 4837 | 10.5906 | 10.8970 |
| 10.7162 | 10.8279 | 10 9395 | 11.0511 | 11.1627 |
| 11 1627 | 11.2790 | 11.3953 | 11.5116 | 11.6279 |
| 11.6092 | 11 7302 | 11 8351 | 11.9720 | 12 0930 |
| 12.0558 | 12.1813 | 12.3069 | 12.4325 | 12.6581 |
| 12.5023 | 12 6325 | 12,7627 | 12.8930 | 13.0232 |
| 12 9488 | 13 0837 | 13.2186 | 13 3534 | 13.4883 |
| 13 3952 | 13 5348 | 13.6744 | 13.8139 | 14.9534 |
| .0446 | .0451 | 0455 | 0460 | 0405 |
| 0893 | .0902 | 0911 | .0921 | 0930 |
| .1339 | .1353 | .1367 | .1318 | .1395 |
| .1786 | .1804 | .1823 | .1842 | 1860 |
| .2232 | .2255 | .2279 | 2302 | 2325 |
| .2679 | .2707 | .2735 | .2762 | .2790 |
| .3125 | .3158 | 3190 | .3223 | 3255 |
| .3572 | .3609 | .3646 | 36 3 | 3720 |
| .4018 | .4060 | .4102 | .4144 | .4186 |

| | 101 | 102 | 103 | 104 | 105 |
|---|---|---|---|---|---|
| 30 | 1,4093 | 1,4232 | 1,4372 | 1,4511 | 1,4651 |
| 40 | 1,8790 | 1,8976 | 1,9162 | 1,9348 | 1,9535 |
| 50 | 2,3488 | 2,3720 | 2,3958 | 2,4186 | 2,4418 |
| 60 | 2,8186 | 2,8465 | 2,8744 | 2,9023 | 2 9302 |
| 70 | 3,2883 | 3,3209 | 3,3354 | 3 3866 | 3,4189 |
| 80 | 3,7581 | 3,7953 | 3,8325 | 3,8697 | 3,9069 |
| 90 | 4,2279 | 4 2697 | 4.3119 | 4,3534 | 4 3953 |
| 100 | 4.6976 | 4,7441 | 4.7909 | 4.8372 | 4 8837 |
| 110 | 5,1674 | 5,2180 | 5,2697 | 5,3209 | 5,3720 |
| 120 | 5,6372 | 5,6930 | 5,7488 | 5,8046 | 5,8604 |
| 130 | 6 1069 | 6,1674 | 6,2279 | 6,2883 | 6,3488 |
| 140 | 6,5767 | 6,6418 | 6 7069 | 9.7720 | 6.8372 |
| 150 | 7,0465 | 7,1102 | 7,1860 | 7,2550 | 7,3256 |
| 160 | 7,5162 | 7,5906 | 7,6651 | 7,7395 | 7,8139 |
| 170 | 7,9860 | 8,0651 | 8.1441 | 8,2232 | 8,3023 |
| 180 | 8,4558 | 8.5395 | 8,6232 | 8,7069 | 8,7907 |
| 190 | 8,9255 | 9,0139 | 9,1023 | 9.1906 | 9,2790 |
| 200 | 9,3953 | 9,4883 | 9,5813 | 9,6744 | 9,7674 |
| 210 | 9,8651 | 9,9627 | 10,0050 | 10,1581 | 10,2558 |
| 220 | 10,3349 | 10 4372 | 10,5395 | 10 6418 | 10,7442 |
| 230 | 0,8049 | 10,9116 | 11,0186 | 11,1255 | 11,2325 |
| 240 | 11,2744 | 11,3860 | 11,4976 | 11,6093 | 11,7206 |
| 250 | 11,7442 | 11,8604 | 11,9767 | 12,0930 | 12.2093 |
| 260 | 12.2139 | 12,3348 | 12.4558 | 12,5767 | 12,6976 |
| 270 | 12,6837 | 12,8093 | 12,9345 | 13,0604 | 13,1860 |
| 280 | 13,1535 | 13,2837 | 13 4139 | 13,5441 | 13,7445 |
| 290 | 13,6232 | 13,7581 | 13,8930 | 14.0279 | 14,1627 |
| 300 | 14,0930 | 14 2325 | 14,3721 | 14,5116 | 14,6511 |
| 1 | ,0469 | 0474 | ,0479 | ,0483 | ,0488 |
| 2 | ,0939 | 0948 | ,0958 | ,0967 | ,0976 |
| 3 | ,1409 | ,1423 | ,1437 | ,1451 | ,1465 |
| 4 | ,1879 | ,1897 | ,1916 | ,1935 | .1953 |
| 5 | ,2348 | ,2372 | ,2395 | ,2418 | ,2442 |
| 6 | ,2818 | ,2846 | ,2874 | ,2902 | ,2930 |
| 7 | ,3288 | ,3321 | ,3153 | 3386 | .3418 |
| 8 | .3758 | .3795 | ,3832 | .3869 | ,3907 |
| 9 | ,4228 | ,4269 | ,4311 | ,4353 | ,4395 |

| 106 | 107 | 108 | 109. | 110 |
|---|---|---|---|---|
| 1.4790 | 1.4930 | 1.5062 | 1.5209 | 1.5349 |
| 1.9721 | 1 9907 | 2.0092 | 2.0278 | 2 0465 |
| 2.4651 | 2.4883 | 2.5116 | 2.5348 | 2.5581 |
| 2.9581 | 2.9860 | 3.0139 | 3.0418 | 3.0697 |
| 3.4511 | 3.4837 | 3.5162 | 3 5488 | 3.5814 |
| 3.9442 | 3.9814 | 4.0186 | 4.0558 | 4.0930 |
| 4.4372 | 4.4790 | 4.5209 | 4 5628 | 4.6046 |
| 4.9302 | 4.9767 | 5.023 | 5.0697 | 5.1163 |
| 5.4232 | 5.4744 | 5.5255 | 5.5767 | 5 6279 |
| 5 9163 | 5.9721 | 6.0279 | 6.0837 | 6.1395 |
| 6.4093 | 6.4697 | 6.5302 | 6.5907 | 6.6511 |
| 6.9023 | 6.9674 | 7.0325 | 7.0976 | 7.1628 |
| 7.3953 | 7.4651 | 7.5348 | 7.6046 | 7.6744 |
| 7.8883 | 7.9628 | 8 0372 | 8.1116 | 8.1860 |
| 8.3814 | 8 4604 | 8.5395 | 8.6186 | 8.6976 |
| 8.8744 | 8 9581 | 9.0418 | 9.1256 | 9.2093 |
| 9.3674 | 9 4558 | 9 5442 | 9 6325 | 9.7209 |
| 9.8604 | 9 9534 | 10.0465 | 10.1395 | 10.2325 |
| 10.3534 | 10.4511 | 10.5488 | 10.6465 | 10 7442 |
| 10.8465 | 10.9488 | 11.0511 | 11.1535 | 11.2558 |
| 11.3395 | 11.4465 | 11.5535 | 11.6604 | 11.7674 |
| 11.8325 | 11 9442 | 12.0558 | 12.1674 | 12.2790 |
| 12.3255 | 12.4418 | 12 5581 | 12.6744 | 12.7907 |
| 12.8186 | 12.9395 | 13.0604 | 13.1814 | 13.3023 |
| 12.3116 | 13.4372 | 13.5628 | 13.6883 | 13.8139 |
| 13.8046 | 13.9348 | 14 0651 | 14 1953 | 14 3256 |
| 14.2976 | 14.4325 | 14.5674 | 14.7023 | 14 8372 |
| 14.7007 | 14 9302 | 15.0607 | 15 2003 | 15 3488 |
| .0493 | .0978 | 0502 | 0507 | 0511 |
| 0986 | 0995 | .1004 | .1014 | 1023 |
| .1479 | .1493 | .1506 | .1521 | 1535 |
| 1972 | .1001 | .2009 | .2028 | .2046 |
| .2465 | .2488 | .2511 | .2535 | .2558 |
| .2958 | .2986 | .3014 | .3042 | 3069 |
| .3451 | .3483 | .3516 | 3549 | .3581 |
| .3944 | .3981 | 4018 | .4056 | 4093 |
| .4437 | 4479 | .4521 | 4563 | 60 |

| | 111 | 112 | 113 | 114 | 115 |
|---|---|---|---|---|---|
| 30 | 1,5488 | 1,5628 | 1,5767 | 1,4906 | 1,6046 |
| 40 | 2,0651 | 2,0837 | 2,1023 | 2,1209 | 2,1395 |
| 50 | 2,5814 | 2,6046 | 2,6279 | 2,6511 | 2,6744 |
| 60 | 3.0976 | 3,1256 | 3,1534 | 3,1813 | 3,2093 |
| 70 | 3,6139 | 3,6465 | 3,6790 | 3,7110 | 3 7441 |
| 80 | 4,1302 | 4,1674 | 4,2046 | 4,2418 | 4 2790 |
| 90 | 4,6465 | 4,6883 | 4.7302 | 4,7720 | 4.8139 |
| 100 | 5.1628 | 5,2093 | 5,2558 | 5,3023 | 5,3488 |
| 110 | 5,6790 | 5,7303 | 5.7813 | 5,8325 | 5,8837 |
| 120 | 6,1953 | 6,2511 | 6,3069 | 6.3627 | 6,4186 |
| 130 | 6,7116 | 6,7721 | 6,8325 | 5,8930 | 6,9534 |
| 140 | 7,2279 | 7,2930 | 7,3581 | 7,4232 | 7,4883 |
| 150 | 7.7442 | 7,8139 | 7,8137 | 7,9534 | 8,0232 |
| 160 | 8,2604 | 8,3348 | 8,4093 | 8,4837 | 8,5581 |
| 170 | 8,7767 | 8,8558 | 8,9348 | 9,0139 | 9 0930 |
| 180 | 9,2930 | 9,3767 | 94604 | 9.5441 | 9.6279 |
| 190 | 9,8093 | 9,8976 | 9,9860 | 10,0744 | 10,1627 |
| 200 | 10,3256 | 10,4186 | 10,5116 | 10,6046 | 10,6976 |
| 210 | 10,8418 | 10,9395 | 11,0372 | 11,1348 | 11,2325 |
| 220 | 11,3581 | 11,4604 | 11.5627 | 11,6651 | 11,7674 |
| 230 | 11,8744 | 11,9814 | 12,0883 | 12,1953 | 12,3023 |
| 240 | 12,3906 | 12,5023 | 12,6139 | 12,7255 | 12,8372 |
| 250 | 12,9069 | 13,0232 | 13,1395 | 13,2558 | 13,3720 |
| 260 | 13,4232 | 13 5442 | 13,6651 | 13,7860 | 13,9069 |
| 270 | 13,9395 | 14,0651 | 14,1906 | 14,3162 | 14,4418 |
| 280 | 14,4458 | 14,5860 | 14,7162 | 14.8465 | 14,9767 |
| 290 | 14,9721 | 15,1069 | 15,2418 | 15,3767 | 15,5116 |
| 300 | 15,4883 | 15,6279 | 15 7674 | 15,9069 | 15,0465 |
| 1 | ,0516 | ,0521 | 0525 | ,0530 | ,0535 |
| 2 | ,1032 | ,1042 | ,1051 | ,1060 | ,1069 |
| 3 | ,1548 | .1562 | 1576 | .1590 | ,1604 |
| 4 | ,2065 | 2083 | .2102 | 2121 | ,2136 |
| 5 | ,2581 | 2604 | ,2628 | ,2651 | .2674 |
| 6 | ,3097 | 3125 | 3125 | ,3181 | ,3209 |
| 7 | ,3614 | 3646 | ,3679 | ,3711 | ,3744 |
| 8 | ,4130 | ,4167 | ,4204 | ,4242 | ,4279 |
| 9 | ,4646 | ,4688 | 4772 | ,4730 | ,4814 |

| 116 | 117 | 118 | 119 | 120 |
|---|---|---|---|---|
| 1,6186 | 1,6325 | 1,6465 | 1,6604 | 1,0744 |
| 2,1581 | 2,1767 | 2,1953 | 2,2139 | 2,2325 |
| 2,6976 | 2,7209 | 2,7411 | 2,7676 | 2,7906 |
| 3,2372 | 3,2651 | 3,2930 | 3.3209 | 3,3488 |
| 3.7767 | 3,8093 | 3.8418 | 3,8744 | 3,9069 |
| 4,3162 | 4,3534 | 4,3906 | 4,4279 | 4,4651 |
| 4,8558 | 4 8976 | 4,6395 | 4,9813 | 5 0232 |
| 5.3953 | 5,4418 | 5 4883 | 5.5348 | 5.5813 |
| 5.9348 | 5,9860 | 6,0372 | 6,0883 | 6,1395 |
| 6,4744 | 6,5302 | 6 5860 | 6.6418 | 6,6976 |
| 7,0139 | 7,0744 | 7,1348 | 7,1953 | 7,2558 |
| 7.5534 | 7,6186 | 7,6837 | 7,7488 | 7,8139 |
| 8 0930 | 8,1627 | 8,2825 | 8,3023 | 8,3720 |
| 8,6325 | 8,7069 | 8,7813 | 8,8558 | 8,9302 |
| 9,1720 | 9,2511 | 9,3302 | 9,4093 | 9,4883 |
| 9,7116 | 9,7953 | 9,8790 | 9,9627 | 10,0465 |
| 10,2511 | 10,3395 | 10,4280 | 10,5162 | 10,6046 |
| 10,7906 | 10,8837 | 10,9767 | 11,0697 | 11,1627 |
| 11,3302 | 11,4279 | 11,5255 | 11,6232 | 11,7209 |
| 11.8697 | 11,9720 | 12,0744 | 12,1767 | 12,2790 |
| 12,4093 | 12,5162 | 12,6234 | 12,7302 | 12,8372 |
| 12,9488 | 13,0604 | 13 1720 | 13,2837 | 13,3953 |
| 13,4883 | 13,6046 | 13,7219 | 13,8372 | 13,9534 |
| 14,0279 | 14,1148 | 14,2696 | 14,3906 | 14,5116 |
| 14,5674 | 14.6930 | 14,8180 | 14.9441 | 15,0697 |
| 15,1069 | 15,2372 | 15,3674 | 15,4976 | 15,6279 |
| 15,6465 | 15,7813 | 15,9164 | 16,0511 | 16,1860 |
| 16,1860 | 16,3255 | 16,4651 | 16,6046 | 16,7441 |
| ,0539 | ,0544 | ,0548 | ,0553 | 0558 |
| 1079 | .1088 | ,1097 | .1107 | ,1116 |
| .1618 | ,1632 | ,1646 | .1660 | ,1674 |
| ,2158 | 2176 | ,2195 | .2214 | ,2232 |
| 2697 | ,2721 | ,2744 | ,2767 | .2790 |
| ,3237 | ,3265 | .3293 | ,3321 | 3348 |
| ,3776 | .3809 | ,3842 | .3874 | 3907 |
| ,4316 | ,4353 | ,4390 | ,4428 | ,4465 |
| ,4855 | ,4897 | .4629 | ,4081 | ,4023 |

## *To gauge a* Couch *which lies in the Form of a Parallelogram.*

### *RULE.*

Find the Breadth in the Top of the Table, and the Length in the firſt Column, and in the Angle of Meeting, you'll have the Content in Malt Buſhels at one Inch Deep.

### *EXAMPLE* I.

Let a Couch be propoſed, whoſe Length is 199 Inches, and Breadth 99; and the Area be required.

Look for 99 in the Top of the Table, and under that, and againſt 190 you'll find 8,7488
and againſt 9, ,4144

their Sum is 9,1632
The Area required; which if you multiply by the Depth, produceth the whole Content in Malt Buſhels.

### *EXAMPLE* II.

Admit the Length of a Couch to be 559 Inches, and Breadth 342; what's the Area?

1. Look for 100 at the Top, and under it,
and againſt 300 you'll find 13,9534
againſt 200 you'll find 9,3623
againſt 50 you'll find 2,2358
againſt 9 you'll find 4186
their Sum is 26,0001
26,0001
and ſo much is the Area of 559 Inches 26,0001
long and 100 broad; but becauſe 'tis 78,0003
300, ſet it down as in the Margin.

2. Look 42 in the Top, and under it, and
againſt 300 you'll find 5,8604
againſt 200 you'll find 3,9069
againſt 50 you'll find ,9767
and againſt 9 you'll find ,1758
their Sum is 10,9198
the Area of 559 long, and 42 broad 78,0003

to which add 78,0003, the Area of 559 long, and 300 broad, and the Sum will be 88,9201, which is the Area of a Couch, whoſe Breadth is 342 Inches, and Length 559; and the like may be done by any other which exceeds the Table.

### *To gauge a Triangle by the Table.*

### *RULE* I.

Find half the Perpendicular in the Top of the Table; then under that, and againſt the Baſe you'll have the Area.

EX.

## EXAMPLE II.

Let the Perpendicular of the Triangle propoſed be 86 Inches, and the Baſe 255; what's the Area?

Look in the Top of the Table for 43, and under that, and againſt 250 you'll find 5,0000
and againſt 5, 0,1000
their Sum, *viz.* 5,1000
is the Area required.

I might here proceed, and ſhow how to find the Area's of all other right-lined Figures, and alſo the Contents of ſuch Tuns to which any right-lined Superficies may be a Baſe; but if what is already laid down in this Book, be well underſtood, the Learner will be able to perform any ſuch Queſtion without further Directions.

## *A practical Method of finding the Area of any Couch, without either Pen, Sliding Rule, or Table, as 'twas delivered to me by my ingenious Friend Mr.* Fitz Lambe, *now Superviſor in* Worceſter *Collection.*

The Rules for Couch Gauges are theſe.

If the Length or Breadth be 215, or 2,15, the other is the Area. If

| If | Multiply by |
|---|---|
| 430 or 43 | 2 |
| 64,5 or 645 | 3 |
| 860 | 4 |
| 107,5 | 5 |
| 129 | 6 |
| 150,5 | 7 |
| 172, | 8 |
| 193,5 | 9 |

If the Dimenſions be otherwiſe, it may be performed near the Truth by the Obſervations following.

If the Length and Breadth be near the ſame, take from one and add the like Number to the other; ſo that you make one of them equal to one of the above Numbers.

If the Length be double (or near double) the Breadth, what you take from the Length muſt be but Half, or not quite Half, added to the Breadth: Or if you take from the Breadth, you muſt add Double, or rather more than Double to the Length, and ſo in Proportion, and then proceed as above.

## A TABLE to gauge right-lined Figures; When the Dimenſions are taken in Inches and decimal Parts, as in Ciſterns, *&c.*

|  | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 1 | .000465 | .000516 | .000558 | .000604 | .000651 | .000697 |
| 2 | .000930 | .000976 | .000023 | .001069 | .001116 | .001162 |
| 3 | .001395 | .001442 | .001488 | .001534 | .001581 | .001628 |
| 4 | .001860 | .001900 | .001953 | .002000 | .002046 | .002093 |
| 5 | .002325 | .002372 | .002418 | .002246 | .002511 | .002558 |
| 6 | .002790 | .002837 | .002885 | .002930 | .002976 | .003023 |
| 7 | .003256 | .003302 | .003344 | .003395 | .003442 | .003488 |
| 8 | .003721 | .003767 | .003819 | .003860 | .003907 | .003953 |
| 9 | .004186 | .004232 | .004279 | .004325 | .004372 | .004418 |
| 10 | .004651 | .004697 | .004744 | .004790 | .004837 | .004884 |
| 11 | .005116 | .005163 | .005209 | .005256 | .005027 | .005349 |
| 12 | .005581 | .005628 | .005674 | .005721 | .005767 | .005814 |
| 13 | .006046 | .006093 | .006139 | .006150 | .006232 | .006279 |
| 14 | .006511 | .006558 | .006604 | .006651 | .006697 | .006744 |
| 15 | .006977 | .007023 | .007239 | .007116 | .007163 | .007209 |
| 16 | .007442 | .007488 | .007535 | .007581 | .007628 | .007674 |
| 17 | .007907 | .007953 | .008000 | .008046 | .008093 | .008139 |
| 18 | .008372 | .008418 | .008465 | .008511 | .008558 | .008604 |
| 19 | .008837 | .008883 | .008930 | .008976 | .009023 | .009065 |
| 20 | .009302 | .009348 | .009395 | .009441 | .009435 | .009534 |
| 21 | .009767 | .009813 | .009860 | .009906 | .009953 | .010000 |
| 22 | .010232 | .010279 | .010325 | .010372 | .010418 | .010465 |
| 23 | .010697 | .010744 | .010790 | .010837 | .010883 | .010837 |
| 24 | .011162 | .011209 | .011255 | .011302 | .011348 | .011395 |
| 25 | .011628 | .011674 | .011720 | .011767 | .011813 | .011860 |
| 26 | .012093 | .012139 | .012186 | .012232 | .012279 | .012325 |
| 27 | .012558 | .012604 | .012651 | .012697 | .012744 | .012790 |
| 28 | .013023 | .013069 | .013116 | .013162 | .013209 | .013255 |
| 29 | .013488 | .013534 | .013581 | .013627 | .013675 | .013725 |
| 30 | .013953 | .014000 | .014046 | .014093 | .014139 | .014186 |
| 31 | .014418 | .014465 | .014511 | .014558 | .014604 | .014651 |
| 32 | .014883 | .014930 | .014976 | .015023 | .015069 | .015116 |
| 33 | .015349 | .015395 | .015441 | .015488 | .015535 | .015581 |
| 34 | .015813 | .015860 | .015907 | .015953 | .016000 | .016046 |
| 35 | .016279 | .016325 | .016372 | .016418 | .016465 | .016511 |
| 36 | .016744 | .016790 | .016837 | .016883 | .016930 | .016976 |
| 37 | .017209 | .017255 | .017302 | .017348 | .017395 | .017446 |
| 38 | .017614 | .017720 | .017767 | .017813 | .017860 | .017907 |
| 39 | .018139 | .018186 | .018232 | .018279 | .018325 | .018372 |
| 40 | .018604 | .018651 | .018693 | .018744 | .018790 | .0188[illegible] |
| 41 | .019067 | .019116 | .019162 | .019209 | .019256 | .019302 |
| 42 | .019534 | .019581 | .019627 | .019674 | .019720 | .019767 |
| 43 | .020002 | .020046 | .020093 | .020139 | .020186 | .020232 |

| 6 | 7 | 8 | 9 | A | B |
|---|---|---|---|---|---|
| .000744 | .000790 | .000837 | .000884 | 43 | .02 |
| .001209 | .001255 | .001302 | .001348 | 86 | .04 |
| .001674 | .001721 | .001767 | .001814 | 129 | .06 |
| .002139 | .002186 | .002232 | .002279 | 172 | .08 |
| .002604 | .002651 | .002697 | .002744 | 215 | .10 |
| .003069 | .003116 | .003163 | .003209 | 258 | .12 |
| .003535 | .003581 | .003628 | .003674 | 301 | .14 |
| .004000 | .004046 | .004093 | .004139 | 344 | .16 |
| .004465 | .004511 | .004558 | .004604 | 387 | .18 |
| .004930 | .004977 | .005023 | .005069 | 430 | .20 |
| .005395 | .005442 | .005488 | .005535 | 473 | .22 |
| .005860 | .005907 | .005553 | .006000 | 516 | .24 |
| .006325 | .006372 | .006418 | .006465 | 559 | .26 |
| .006790 | .006837 | .006884 | .006930 | 602 | .28 |
| .007256 | .007302 | .007348 | .007395 | 645 | .30 |
| .007721 | .007767 | .007814 | .007860 | 688 | .32 |
| .008186 | .008232 | .008279 | .008325 | 731 | .34 |
| .028651 | .008607 | .008744 | .008790 | 774 | .36 |
| .009116 | .009162 | .009203 | .009255 | 817 | .38 |
| .009581 | .009627 | .009674 | .009720 | 860 | .40 |
| .010046 | .010093 | .010139 | .010186 | 903 | .42 |
| .010511 | .010558 | .010604 | .010651 | 946 | .44 |
| .010976 | .011023 | .011069 | .011116 | 989 | .46 |
| .011441 | .011488 | .011534 | .011581 | 1032 | .48 |
| .011906 | .011953 | .012000 | .012046 | 1075 | .50 |
| .012372 | .012418 | .012465 | .012511 | 1118 | .52 |
| .012837 | .012883 | .012930 | .012976 | 1161 | .54 |
| .013302 | .013348 | .013395 | 013441 | 1204 | .56 |
| .013767 | .013813 | .013866 | .013906 | 1247 | .58 |
| .014232 | .014279 | .013425 | .014372 | 1290 | .60 |
| .014697 | .014744 | .014790 | .014837 | 1337 | .62 |
| .015162 | .015209 | .015255 | .015302 | 1376 | .64 |
| .015628 | .015674 | .015721 | .015767 | 1419 | .66 |
| .016033 | .016139 | .016186 | 016232 | 1462 | .68 |
| 016558 | .016604 | .016651 | .016697 | 1505 | .70 |
| .017023 | .017069 | .017116 | .017162 | 1548 | .72 |
| .017488 | .017534 | .017581 | 017627 | 1591 | .74 |
| .017953 | .018000 | .018046 | .018093 | 1634 | .76 |
| .018418 | .018465 | .018511 | 018558 | 1677 | .78 |
| .018883 | .018930 | .018976 | .019023 | 1720 | .80 |
| .019349 | .019395 | .019442 | 019488 | 1763 | .82 |
| .019814 | .019860 | .019907 | 019953 | 1806 | .84 |
| 020279 | .020325 | .020372 | .020418 | 1849 | .86 |

## The Uſe of the preceding Table for Ciſtern Gauges, &c.

### CASE I.

If the Length or Breadth of a Rectangular Ciſtern, *&c.* be any Number in the Column under A; multiply the other by the Decimal anſwering thereto under B, and the Product will be the Area at one Inch Deep: Which multiplied by the whole Depth, produceth the whole Content in Malt Buſhels, and decimal Parts.

### *EXAMPLE* I.

Let a Ciſtern be propoſed, whoſe Length is 257 Inches, and Breadth 172; and the Area at one Inch Deep, be required.

Firſt look under A for either of the two Numbers, *viz.* 257, or 172, and there you'll find 172; and againſt it under B, ſtands ,08, by which you are to multiply 257, and the Product will be 20,56, which is the Area required, *i. e,* 20 Buſhels and $\frac{56}{100}$

### *EXAMPLE* II.

Given the Length of a Ciſtern or Couch 989 Inches, and Breadth 771, to find the Area.

Look under A, and there you'll find 989; againſt which under B, you'll find ,46 for a Multiplier; by which multiply 771, and the Product will be 354,66, the Area required.

### CASE II.

If neither the Length or Breadth can be found under A, you muſt take the next Number leſs under either of them, and note what Decimal ſtands againſt it under B; then take the Difference of the Number propoſed, and that next leſs which you find under A, and enter the firſt Column of the Table, and againſt that (and under the Tenths, if any be) you'll find another decimal Number, which added to your before-noted Decimal which you found under B, will give a Multiplier to be uſed as before.

### *EXAMPLE.*

Let the Length be 1158,5, and Breadth 980,8, and the Area be required.

Firſt I look under A, and find not juſt 980,8, but the next to it leſs is 946, againſt which ſtands ,44 under B, which I note; then I take 946 out of 980,8, and the Remains will be 34,8; look 34 in the firſt Column, and againſt it under ,8 you'll find ,0161861, which add to the Decimal before found, *viz.* ,44, and the Sum will be ,456186, by which multiply 1158,5 and the Product will be 528,491, *&c.* the Area required, and ſo of any other: And though I have uſed 6 decimal Places here, yet four will be ſufficient in moſt Caſes. And thus having ſhewn the Uſe of this little, yet moſt extenſive Table; I ſhall next endeavour to ſhew any young Officer, how, when he has many Gauges to caſt up by the Sliding Rule, he may do them with that Inſtrument at one Set without finding the Area, or a Mean firſt, as is common.

*The Way is this.*

Take the Sliding Rule in your Left-hand, and [illegible] Side marked with A. B. num. Seg. ly. towards [illegible] done, with your Right-hand draw out the Sliding-[illegible] and place the Rabbit of the ſame which is next the [illegible] Numbers on the Corner of the Sliding Rule, which is at the Line of *Seg. ly.* ſo as that the 1 at the Beginning of the Line of Numbers on the Sliding-piece noted with B, may be againſt 10 on the Line of Numbers on the Sliding-piece C, or that the End of the Line of Numbers on the Sliding-piece C, may ſtand againſt the Beginning of the Line of Numbers on the Sliding-piece B. When the Piece is ſet as it ought to be, it will be perpendicular to the Side A. B. num. *Seg. ly.* and you may keep it in that Poſition with the Thumb of your Left-hand; and if you hold the Rule ſo, as 'tis now ſuppoſed to be placed, the Part of the Line of Inches which is on the In-ſide of the Sliding-piece C, will be towards you and lower-moſt, and the Area's thereto belonging upper-moſt. When the Sliding-piece C is placed on the Corner of the Rule at the Line of *Seg. ly.* as above directed, ſlip it along with your Right-hand, till 2150, in the Line of Numbers on the ſame ſtands exactly on the Line

marked

[illegible] 100 on the Line of *Seg. ly.* and there keep [illegible] Thumb of your Left-hand placed on the In-[illegible] And as that Piece is now ſet, you may at [illegible] the Piece B, anſwer any ſuch Queſtion as is ſet [illegible]

If a Couch be propoſed, whoſe Length is 89 Inches, Breadth 64, and Depth 22, and its Content be required.

The Rule being ſet as above directed, move with your Right-hand the Sliding-piece B, till 22 (the Depth propoſed) in the Line of Numbers there comes againſt 64 on the Line of Numbers on the Piece C, (which with Uſe you will do very exact,) then againſt 89 on the Line A, you'll find 58, as on the Line B; and ſuch is the Content of that Couch.

### EXAMPLE II.

What's the Content of a Ciſtern, whoſe Length is 89,5 Inches, Breadth 64,2, and Depth 46,5.

The Sliding-piece C remaining in the ſame Poſture, as above, bring 46,5 in the Line of Numbers on the Piece B, [illegible],2 on the Line of Numbers on the Piece C; then againſt 89,5 on the Line A, you'll find 124,27 on B, and ſo much the Ciſtern will contain. And here you are to remember always to ſet the Depth on the Line of Numbers on the Piece B, to either the Length or the Breadth in the Line of Numbers on the Piece C; then againſt the Length or Breadth on the Line A, you'll have the Content required on B: But there may be a Line of Numbers ſo contrived, inſtead of the Line of *Seg. ly.* as to work with thoſe on the Piece B, and anſwer all Queſtions of this Kind very nicely; which I hope will be taken Notice of by ſome ingenious Soul, whoſe Buſineſs it more immediately belongs too. And now having brought to an End what I [illegible] Deſign, and made every Thing (that I thought [illegible] to handle) as plain as poſſible; I hope, I ſhall ([illegible] acceptance of this) be encouraged to appear again [illegible] with ſome Curioſities in the noble Art; which [illegible] ſecretly at preſent conceal.

*FINIS.*

Dodo Dudley Hill

Schoolmaster

Farnborough

Warwickshire

Banbury Oct

28th October 1766

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13

112
256

257

705
7756

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4
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5
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44784

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